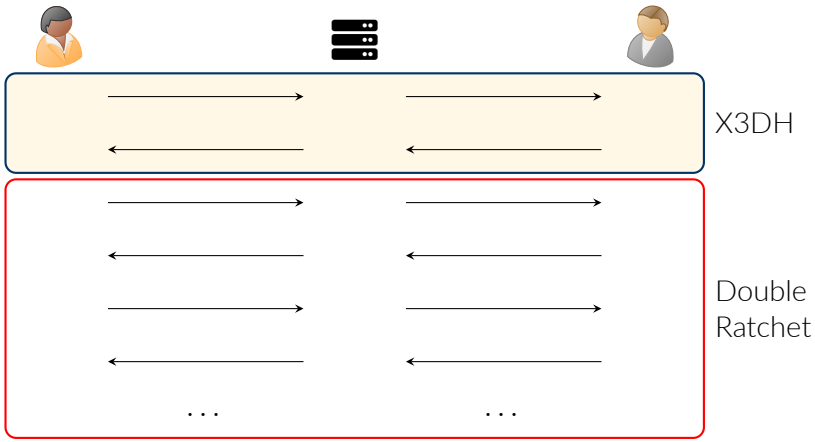
Post-Quantum X3DH

Thomas Prest (joint work with Keitaro Hashimoto,
Shu Katsumata and Kris Kwiatkowski)

PQShield

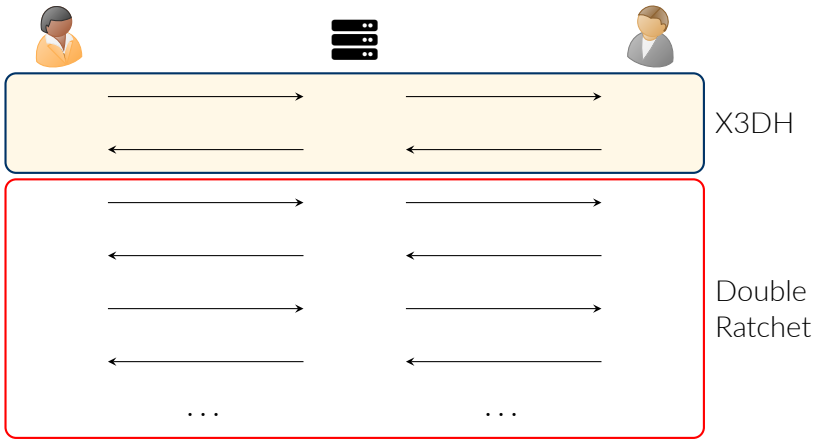
Secure messaging and the Signal protocol



Secure instant messaging:

- Notable differences with TLS: asynchrony, long-lived sessions, etc.
- The security gold standard is the Signal protocol (used in Signal, WhatsApp, etc.)
- Two main components: X3DH and Double Ratchet

Secure messaging and the Signal protocol



How about post-quantum/generic alternatives?

- *For Double Ratchet (generic):* [ACD19]
- *For X3DH (semi-generic):* this talk, based on [HKKP21]

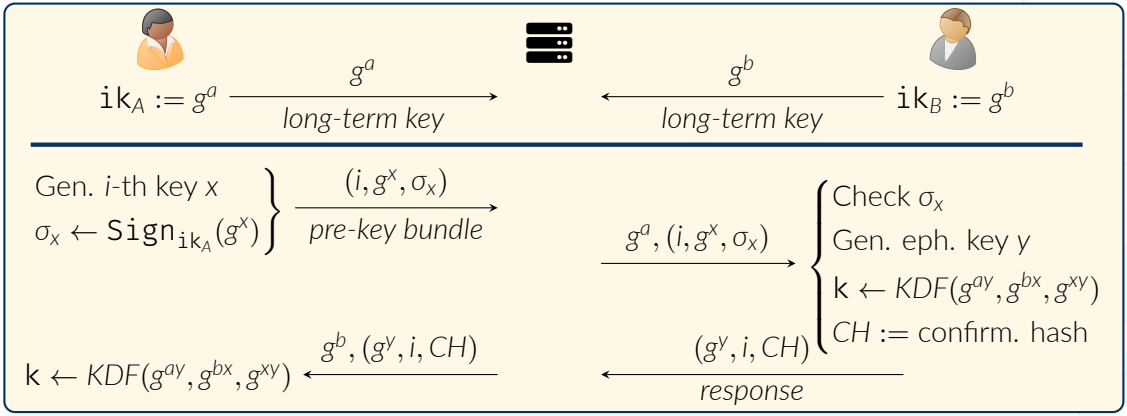
Roadmap

We focus on X3DH:

1. What does it do?
2. What properties do we expect from it?
3. How do we satisfy these properties in a PQ setting?
4. Can we provide a generic construction?

The end result is a PQ alternative (in various flavours) to X3DH.

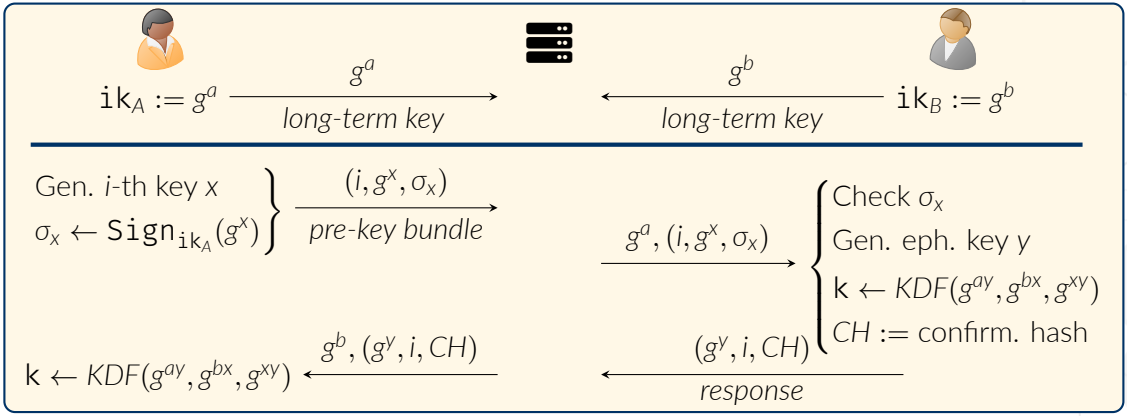
X3DH: reverse-engineering the spec [MP16]



Initial comments:

- Builds upon 3DH (same protocol without σ_x and CH).
- Note the two-message flow and the "*receiver-obliviousness*" of pre-key bundle.
- Parties delete ephemeral keys (x and y) as soon as they can (omitted in figure).

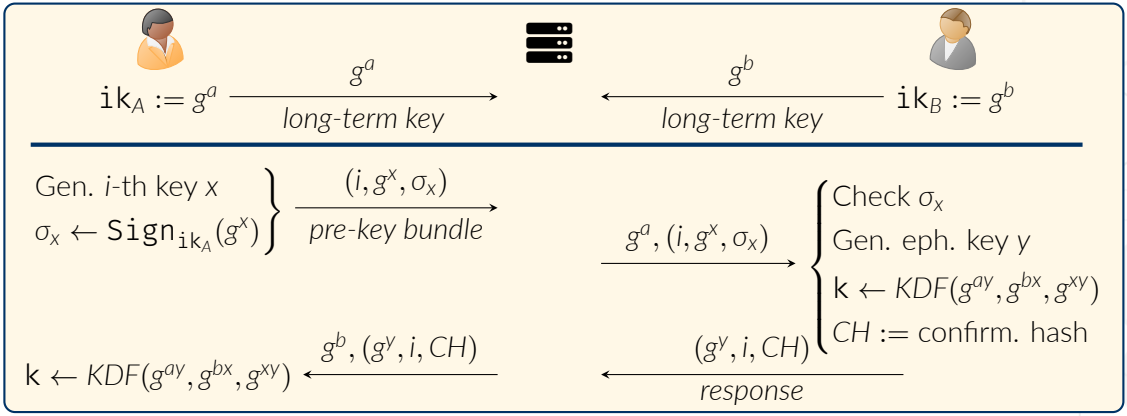
X3DH: reverse-engineering the spec [MP16]



CH is essentially a confirmation hash in disguise [MP16]:

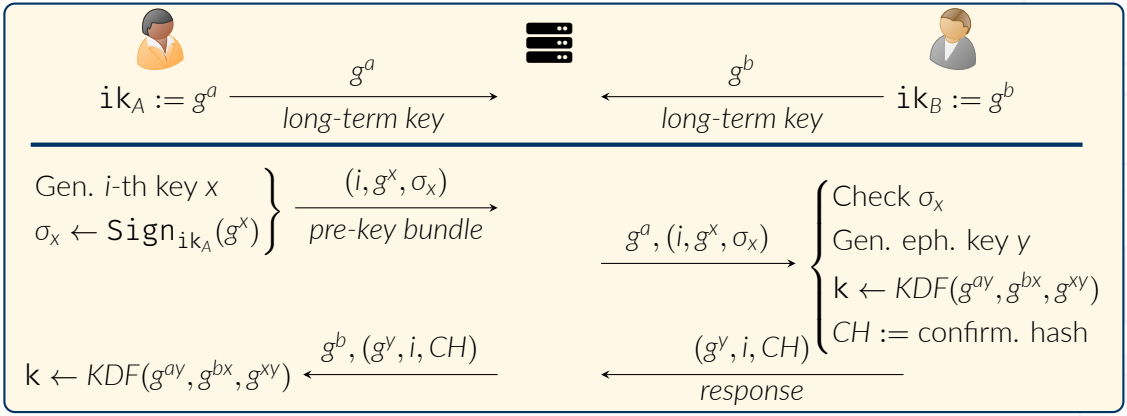
> " [CH is a] ciphertext encrypted with some AEAD encryption scheme using [AD = Encode($\mathtt{ik}_A$) || Encode($\mathtt{ik}_B$)] as associated data and using an encryption key which is either [k or derived from k]. "

X3DH: mutual authentication and forward secrecy



" Note that $[g^{ay}]$ and $[g^{bx}]$ provide mutual authentication, while $[g^{xy}$ provides] forward secrecy. "

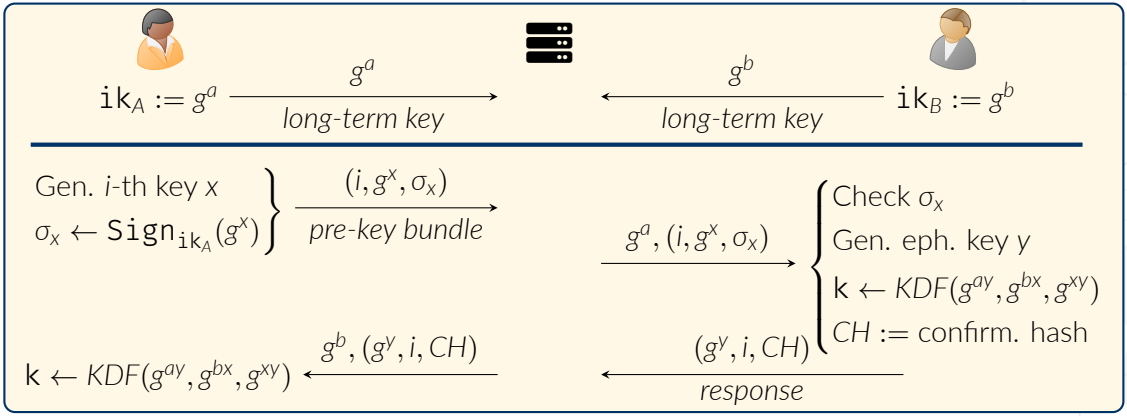
X3DH: mutual authentication and forward secrecy



A few comments on this topic:

- k is hidden as long as adversary doesn't learn $(a \wedge x) \vee (b \wedge y) \vee (x \wedge y)$.
- CH "upgrades" implicit authentication of Bob to explicit.

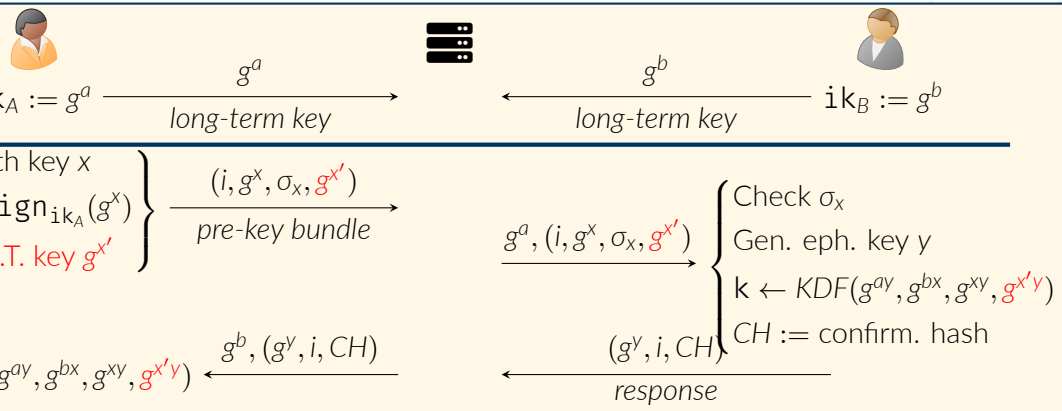
X3DH: deniability



> " X3DH doesn't give either Alice or Bob a publishable cryptographic proof of the contents of their communication or the fact that they communicated. "

- X3DH may only achieve *offline* (as opposed to *online*) deniability [MP16].
- Adversarial model: *semi-honest* (*malicious* under stronger assumptions [VGIK20]).

X3DH: one-time keys (optional)



X3DH allows to provide an optional one-time key.

- *Motivation:* forward secrecy and protection against replay attacks
- *Replay attacks:* there are simpler ways to thwart them
- *Forward secrecy:* largely addressed by signed pre-keys and Double Ratchet

List of desirable properties

Functional properties:

- *Two-message flow:* Initiator → Responder → Initiator
- *Receiver-obliviousness:* first message is independent of the Responder[1]

Security properties:

- *Confidentiality:* session keys looks pseudorandom except for intended parties
  - Weak perfect forward secrecy (wPFS)
  - Perfect forward secrecy (PFS)
- *Mutual authentication:* Initiator knows she talks to Responder, and reciprocally
  - Includes *Key-Compromise Impersonation* (KCI) attacks[2]
  - Includes *Unknown Key-Share* (UKS) attacks[3]
- *Deniability:* ~~online/~~offline, against semi-honest/malicious adversaries

[1] Also known as *post-specified peers*.

[2] KCI: The adversary uses A's long-term key to impersonate other parties towards A.

[3] UKS: A and B compute the same key, but A believe he's talking to C.

How hard is it to devise a PQ alternative to X3DH?

PQ SHIELD

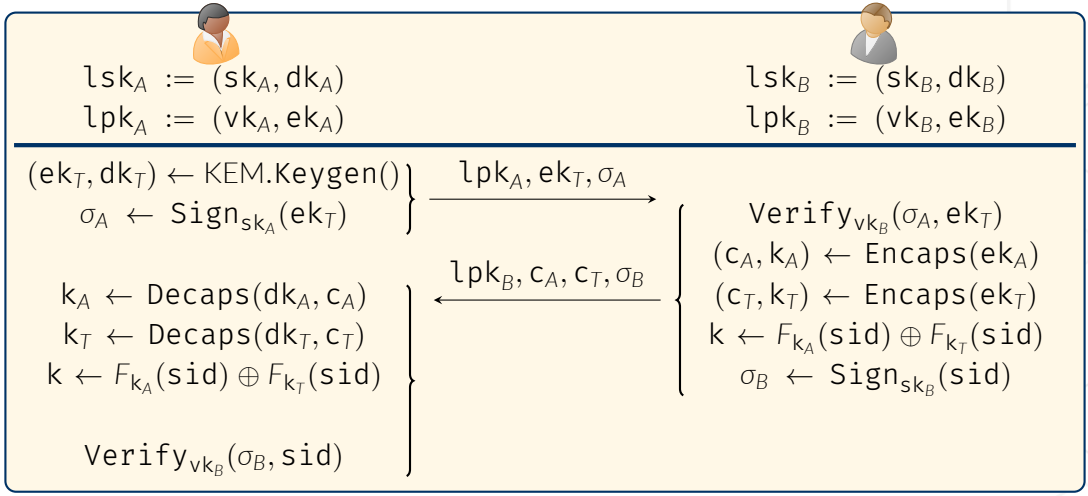
Natural approach #1: replace classical Diffie-Hellman by post-quantum variant.

1. *Noisy Diffie-Hellman* (lattices and codes) [DXL12, Pei14, ADPS16]
   - Doesn't support static keys [Flu16, DFR18]
2. Gap variant of CSIDH (isogenies) [dKsV20, KTAT20, BFG+20]
   - Slow
   - Quantum security of *standard* CSIDH is debated [BLMP19, BS20, Pei20, CCJR20]
3. SIDH and variants thereof (isogenies) [DG21]
   - Uses a SIDH proof of knowledge [FDGZ21] to thwart adaptive attacks [GPST16].
   - The size of the SIDH proof of knowledge needs to be determined.

Natural approach #2: build a protocol that captures as many properties of X3DH as possible using only generic building blocks.

- → KEMs
- → Signatures
- → PRFs
- → etc.

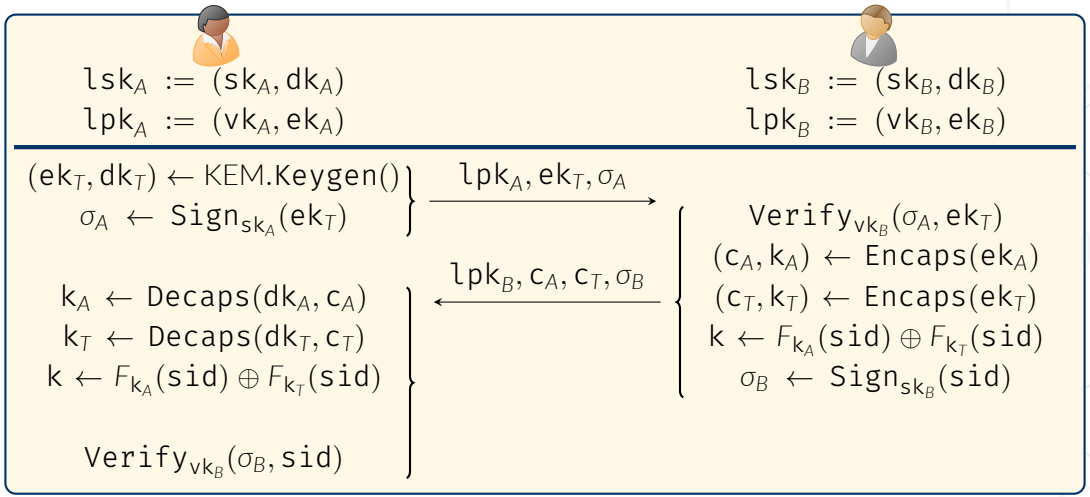
A first generic construction (V0)



Initial comments:

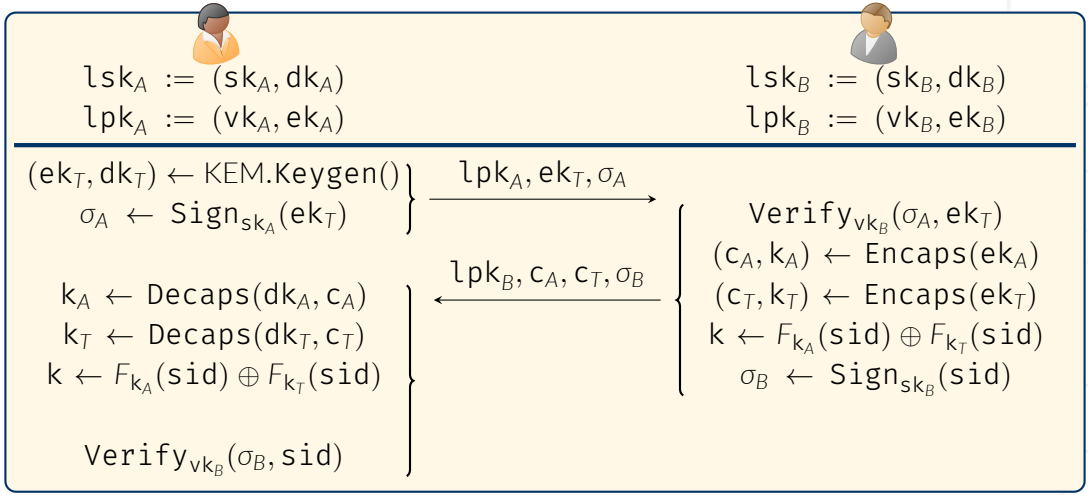
- $\mathtt{sid} = (\mathtt{lpk}_A \| \mathtt{lpk}_B \| \mathtt{ek}_A \| \mathtt{c}_A \| \mathtt{c}_T)$ is the session identifier.
- Omitted for clarity: $\mathtt{k}_A$ and $\mathtt{k}_T$ may be processed by randomness extractors.

A first generic construction (V0)



- *Leakage of long-term keys:* secrecy of $\mathtt{k}$ is guaranteed by $\mathtt{ek}_T$ and $\mathtt{c}_T$
- *State leakage (irrelevant for)*: secrecy of $\mathtt{k}$ is guaranteed by $\mathtt{c}_A$
- *KCI:* prevented by σ_B
- *UKS:* prevented by $\mathtt{sid}$ and $\mathtt{ek}_A$

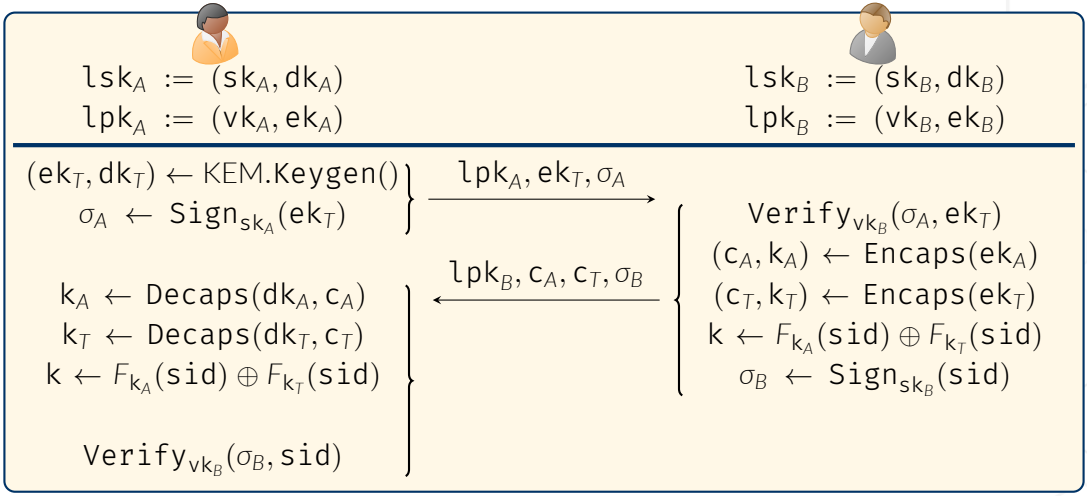
A first generic construction (V0)



→ is explicitly authenticated by his signature.

→ is implicitly authenticated since no one else than her may recover $\mathtt{k}$.

A first generic construction (V0)



Minor possible tweaks:

- *Omit σ_A:* Downgrades PFS to wPFS (note: Double Ratchet then provides PFS again)
- *NAXOS trick:* mitigates leakage of 's randomness by combining it with $\mathtt{lsk}_B$

A very weakly deniable version (V1)

$\mathtt{lsk}_A := (\mathtt{sk}_A, \mathtt{dk}_A)$
$\mathtt{lpk}_A := (\mathtt{vk}_A, \mathtt{ek}_A)$

$\mathtt{lsk}_B := (\mathtt{sk}_B, \mathtt{dk}_B)$
$\mathtt{lpk}_B := (\mathtt{vk}_B, \mathtt{ek}_B)$

$(\mathtt{ek}_T, \mathtt{dk}_T) \leftarrow \mathtt{KEM.Keygen}()$
$\sigma_A \leftarrow \mathtt{Sign}_{\mathtt{sk}_A}(\mathtt{ek}_T)$

$\xrightarrow{\mathtt{lpk}_A, \mathtt{ek}_T, \sigma_A}$

$\mathtt{Verify}_{\mathtt{vk}_B}(\sigma_A, \mathtt{ek}_T)$
$(\mathtt{c}_A, \mathtt{k}_A) \leftarrow \mathtt{Encaps}(\mathtt{ek}_A)$
$(\mathtt{c}_T, \mathtt{k}_T) \leftarrow \mathtt{Encaps}(\mathtt{ek}_T)$
$\mathtt{k} \| \mathtt{k}_\sigma \leftarrow F_{\mathtt{k}_A}(\mathtt{sid}) \oplus F_{\mathtt{k}_T}(\mathtt{sid})$
$\sigma_B \leftarrow \mathtt{Sign}_{\mathtt{sk}_B}(\mathtt{sid})$
$\mathtt{c} \leftarrow \sigma_B \oplus \mathtt{k}_\sigma$

$\xleftarrow{\mathtt{lpk}_B, \mathtt{c}_A, \mathtt{c}_T, \mathtt{c}}$

$\mathtt{k}_A \leftarrow \mathtt{Decaps}(\mathtt{dk}_A, \mathtt{c}_A)$
$\mathtt{k}_T \leftarrow \mathtt{Decaps}(\mathtt{dk}_T, \mathtt{c}_T)$
$\mathtt{k} \| \mathtt{k}_\sigma \leftarrow F_{\mathtt{k}_A}(\mathtt{sid}) \oplus F_{\mathtt{k}_T}(\mathtt{sid})$
$\sigma_B \leftarrow \mathtt{c} \oplus \mathtt{k}_\sigma$
$\mathtt{Verify}_{\mathtt{vk}_B}(\sigma_B, \mathtt{sid})$

We can tweak the protocol to achieve a weak flavour of deniability for free.

- We mask σ_B using a pseudorandom keystream derived from $\mathtt{k}_A, \mathtt{k}_T$ (tangerine).
- The transcript makes  anonymous as long as doesn't cooperate.
- However if leaks σ_B, then is bound.

A weakly deniable version (V1.5)

$\mathtt{lsk}_A := (\mathtt{sk}_A, \mathtt{dk}_A)$
$\mathtt{lpk}_A := (\mathtt{vk}_A, \mathtt{ek}_A)$

$\mathtt{lsk}_B := (\mathtt{sk}_B, \mathtt{dk}_B)$
$\mathtt{lpk}_B := (\mathtt{vk}_B, \mathtt{ek}_B)$

$(\mathtt{ek}_T, \mathtt{dk}_T) \leftarrow \mathtt{KEM.Keygen}()$
$\sigma_A \leftarrow \mathtt{Sign}_{\mathtt{sk}_A}(\mathtt{ek}_T)$

$\xrightarrow{\mathtt{lpk}_A, \mathtt{ek}_T, \sigma_A}$

$\mathtt{Verify}_{\mathtt{vk}_B}(\sigma_A, \mathtt{ek}_T)$
$(\mathtt{c}_A, \mathtt{k}_A) \leftarrow \mathtt{Encaps}(\mathtt{ek}_A)$
$(\mathtt{c}_T, \mathtt{k}_T) \leftarrow \mathtt{Encaps}(\mathtt{ek}_T)$
$\mathtt{k} \| \mathtt{k}_\sigma \leftarrow F_{\mathtt{k}_A}(\mathtt{sid}) \oplus F_{\mathtt{k}_T}(\mathtt{sid})$
$\sigma_B \leftarrow \mathtt{RS.Sign}_{\mathtt{sk}_B, \mathtt{ring}}(\mathtt{sid})$
$\mathtt{c} \leftarrow \sigma_B \oplus \mathtt{k}_\sigma$

$\xleftarrow{\mathtt{lpk}_B, \mathtt{c}_A, \mathtt{c}_T, c}$

$\mathtt{k}_A \leftarrow \mathtt{Decaps}(\mathtt{dk}_A, \mathtt{c}_A)$
$\mathtt{k}_T \leftarrow \mathtt{Decaps}(\mathtt{dk}_T, \mathtt{c}_T)$
$\mathtt{k} \| \mathtt{k}_\sigma \leftarrow F_{\mathtt{k}_A}(\mathtt{sid}) \oplus F_{\mathtt{k}_T}(\mathtt{sid})$
$\sigma_B \leftarrow \mathtt{c} \oplus \mathtt{k}_\sigma$
$\mathtt{RS.Verify}_{\mathtt{ring}}(\sigma_B, \mathtt{sid})$

- Main idea: replace signatures by *ring* signatures (pine green).
- Setting $\mathtt{ring} = \{\mathtt{vk}_A, \mathtt{vk}_B\}$ only gives a weak flavour of deniability.

A deniable version (offline, semi-honest) (V2)

$\mathtt{lsk}_A := (\mathtt{sk}_A, \mathtt{dk}_A)$
$\mathtt{lpk}_A := (\mathtt{vk}_A, \mathtt{ek}_A)$

$\mathtt{lsk}_B := (\mathtt{sk}_B, \mathtt{dk}_B)$
$\mathtt{lpk}_B := (\mathtt{vk}_B, \mathtt{ek}_B)$

$(\mathtt{ek}_T, \mathtt{dk}_T) \leftarrow \mathsf{KEM.Keygen}()$
$\sigma_A \leftarrow \mathtt{Sign}_{\mathtt{sk}_A}(\mathtt{ek}_T)$
$(\mathtt{sk}_T, \mathtt{vk}_T) \leftarrow \mathsf{RS.Keygen}()$

$\xrightarrow{\mathtt{lpk}_A, \mathtt{ek}_T, \sigma_A, \mathtt{vk}_T}$

$\mathtt{Verify}_{\mathtt{vk}_B}(\sigma_A, \mathtt{ek}_T)$
$(\mathtt{c}_A, \mathtt{k}_A) \leftarrow \mathtt{Encaps}(\mathtt{ek}_A)$
$(\mathtt{c}_T, \mathtt{k}_T) \leftarrow \mathtt{Encaps}(\mathtt{ek}_T)$
$\mathtt{k} \| \mathtt{k}_\sigma \leftarrow F_{\mathtt{k}_A}(\mathtt{sid}) \oplus F_{\mathtt{k}_T}(\mathtt{sid})$
$\sigma_B \leftarrow \mathsf{RS.Sign}_{\mathtt{sk}_B, \mathtt{ring}}(\mathtt{sid})$
$\mathtt{c} \leftarrow \sigma_B \oplus \mathtt{k}_\sigma$

$\xleftarrow{\mathtt{lpk}_B, \mathtt{c}_A, \mathtt{c}_T, \mathtt{c}}$

$\mathtt{k}_A \leftarrow \mathtt{Decaps}(\mathtt{dk}_A, \mathtt{c}_A)$
$\mathtt{k}_T \leftarrow \mathtt{Decaps}(\mathtt{dk}_T, \mathtt{c}_T)$
$\mathtt{k} \| \mathtt{k}_\sigma \leftarrow F_{\mathtt{k}_A}(\mathtt{sid}) \oplus F_{\mathtt{k}_T}(\mathtt{sid})$
$\sigma_B \leftarrow \mathtt{c} \oplus \mathtt{k}_\sigma$
$\mathsf{RS.Verify}_{\mathtt{ring}}(\sigma_B, \mathtt{sid})$

- Main idea: replace signatures by *ring* signatures (pine green).
- ~~Setting $\mathtt{ring} = \{\mathtt{vk}_A, \mathtt{vk}_B\}$ only gives a weak flavour of deniability.~~
- Setting $\mathtt{ring} = \{\mathtt{vk}_T, \mathtt{vk}_B\}$ with $\mathtt{vk}_T$ an ephemeral key makes (V2) truly deniable.

What about malicious adversaries?

Our protocols (V0-V2) are not deniable against *malicious* adversaries.
Example with the Raptor ring signature sheme:

Raptor [LAZ19] (≈ [RST01] + Falcon [PFH+17])

- Each public keys is a polynomial $\mathtt{vk}_i \in \mathbb{Z}_q[x]/(\varphi)$.
- A ring signature of $\mathtt{msg}$ for the ring $(\mathtt{vk}_i)_{i\in\mathcal{R}}$ consists of:
  - a salt $\mathtt{salt}$,
  - a short tuple $s_0 \| (s_i)_{i\in\mathcal{R}}$ such that:

$$s_0 + \sum_{i\in\mathcal{R}} s_i \cdot \mathtt{vk}_i = H(\mathtt{msg}, \mathtt{salt}) \qquad (\bigstar)$$

 can break  's deniability by setting her ephemeral ring signing key maliciously:

1. If $(\mathtt{vk}_T = 0)$, then $(\bigstar)$ becomes: $s_0 + s_B \cdot \mathtt{vk}_B = H(\mathtt{msg}, \mathtt{salt})$
2. If $(\mathtt{vk}_T = x^k)$, then $(\bigstar)$ becomes: $(s_0 + x^k \cdot s_T) + s_B \cdot \mathtt{vk}_B = H(\mathtt{msg}, \mathtt{salt})$

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See our paper for a more theoretical argument likely to encompass more (lattice-based) ring signature schemes.

A few more comments about deniability

Signature sizes for 2-users ring signatures vs standard signatures:

- *NTRU-based:* 2.5 KiB [LAZ19] vs 0.67 KiB [PFH+20]
- *M-LWE/SIS-based:* 4.4 KiB [YEL+21] vs 2.3 KiB [LDK+20]
- *CSIDH-based:* 3.5 KiB [BKP20] vs 0.26 KiB [BKV19]

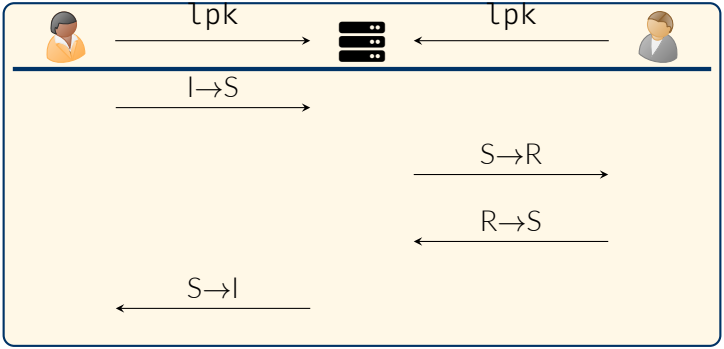
We can upgrade (V2) to a construction (V3) deniable against malicious adversaries:

- *Solution:* 🤦 sends a NIZK proof that she generated vk_T honestly
- *Open question:* can we do that efficiently?

A recent paper [BFG+21] has a construction similar to (V2), but replaces ring signatures with designated-verifier signatures (DVS).

- [BFG+21] shows (Ring signatures $\Rightarrow$ DVS).
- We show that (DVS $\Rightarrow$ Ring signatures), so both are equivalent.

Instantiation of (V1) at NIST level I (≈ AES-128)



Scheme	CPU cycles	lpk	I→S	S→R	R→S	S→I	Total
Falcon/Saber	4.2 MC	1569 B	1362 B	2931 B	2162 B	3731 B	10186 B
Falcon/SIKE	2614 MC	1227 B	1020 B	2247 B	1382 B	2609 B	7258 B
Dilithium/NTRU	6.9 MC	2011 B	3119 B	5130 B	3818 B	5829 B	17896 B
SPHINCS/Saber	269 MC	704 B	17760 B	18464 B	18560 B	19264 B	74048 B

Questions?

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