f-Divergences in Statistical Security Proofs

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The statistical distance

Given two distributions $\boldsymbol{P}, \boldsymbol{Q}$ over Ω, the statistical distance between them is:

$$\Delta_{\mathrm{SD}}(\boldsymbol{P}; \boldsymbol{Q}) = \frac{1}{2} \sum_{x \in \Omega} |\boldsymbol{P}(x) - \boldsymbol{Q}(x)|$$

Useful properties:

1. **Data-processing inequality:** Given a (randomised) function $g : \mathbb{R} \to \mathbb{R}$,

$$\Delta_{\mathrm{SD}}(g(\boldsymbol{P}); g(\boldsymbol{Q})) \leq \Delta_{\mathrm{SD}}(\boldsymbol{P}; \boldsymbol{Q}).$$

2. **Probability-preservation property (PPP):** Given an arbitrary event $E \subseteq \Omega$:

$$|\boldsymbol{P}(E) - \boldsymbol{Q}(E)| \leq \Delta_{\mathrm{SD}}(\boldsymbol{P}; \boldsymbol{Q}).$$

3. **Tensorisation (sub-additivity):** $\Delta_{\mathrm{SD}}(\prod_i \boldsymbol{P}_i; \prod_i \boldsymbol{Q}_i) \leq \sum_i \Delta_{\mathrm{SD}}(\boldsymbol{P}_i; \boldsymbol{Q}_i)$
4. **Triangle inequality:** $\Delta_{\mathrm{SD}}(\boldsymbol{P}; \boldsymbol{R}) \leq \Delta_{\mathrm{SD}}(\boldsymbol{P}; \boldsymbol{Q}) + \Delta_{\mathrm{SD}}(\boldsymbol{Q}; \boldsymbol{R})$
5. **Symmetry:** $\Delta_{\mathrm{SD}}(\boldsymbol{Q}; \boldsymbol{P}) = \Delta_{\mathrm{SD}}(\boldsymbol{P}; \boldsymbol{Q})$

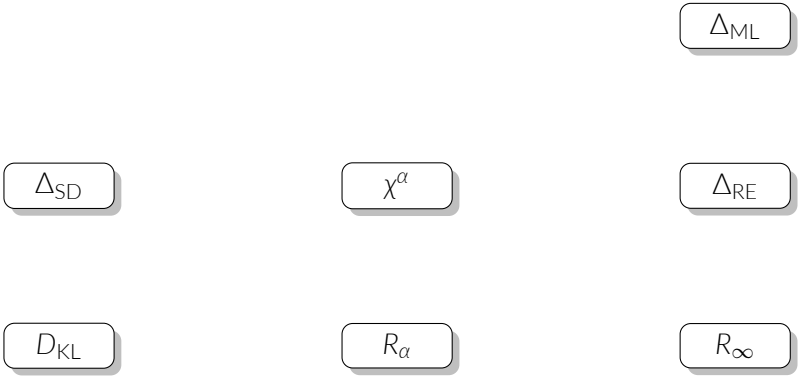
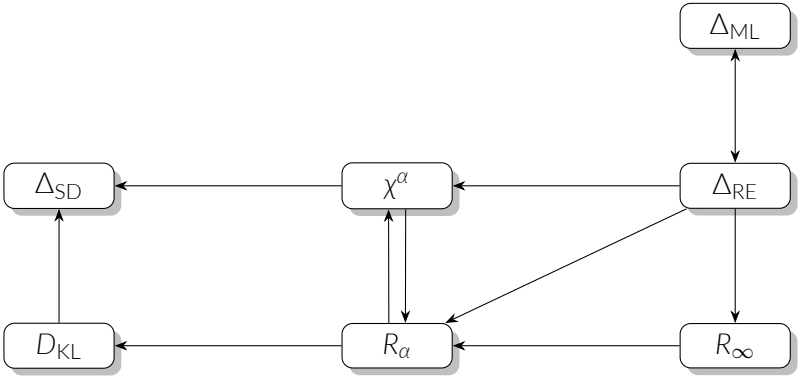


Figure 1: Relations between statistical distance (Δ_{SD}), Kullback-Leibler divergence (D_{KL}), χ^{α} divergence [Vaj73], Rényi divergence (R_{α}), max-log distance (Δ_{ML}), and relative error (Δ_{RE}).



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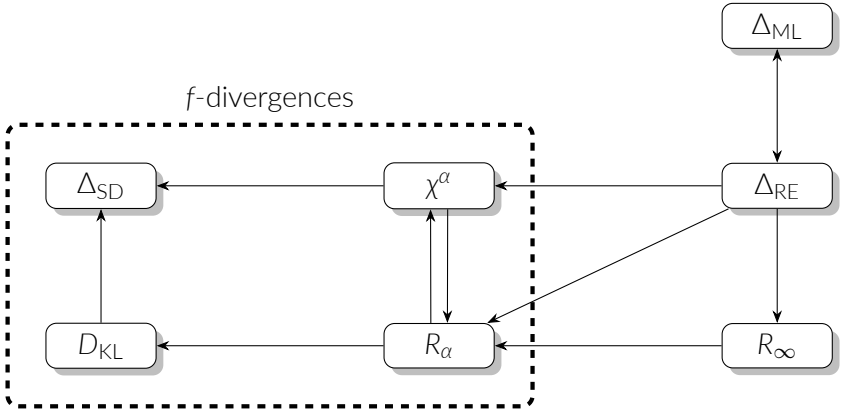

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What are f-divergences?

f-divergence [Mor63, Csi63, AS66]

Let $f : \mathbb{R} \to \mathbb{R}$ be a convex function such that $f(1) = 0$.
The f-divergence between $\boldsymbol{P}$ and $\boldsymbol{Q}$ is:

$$\mathrm{Div}_f(\boldsymbol{P}; \boldsymbol{Q}) = \mathbb{E}_{\boldsymbol{Q}}\left[f\left(\frac{\boldsymbol{P}}{\boldsymbol{Q}}\right)\right] = \sum_{x \in \mathrm{Supp}\,\boldsymbol{Q}} f\left(\frac{\boldsymbol{P}}{\boldsymbol{Q}}\right) \tag{1}$$

Examples

- Statistical distance $\Delta_{\mathrm{SD}}(\boldsymbol{P};\boldsymbol{Q})$
  - $f(x) = \frac{1}{2}|x-1|$
  - Ubiquitous in crypto
- Kullback-Leibler divergence $D_{\mathrm{KL}}(\boldsymbol{P};\boldsymbol{Q})$
  - $f(x) = x \ln x$
  - Ubiquitous in information theory, common in crypto
- Hellinger distance $H^2(\boldsymbol{P};\boldsymbol{Q})$
  - $f(x) = (\sqrt{t}-1)^2$
  - Probabilistic analog of the Euclidean distance
- Rényi divergence $R_\alpha(\boldsymbol{P};\boldsymbol{Q})$
  - $R_\alpha^{\alpha-1} - 1$ is an f-divergence for $f(x) = x^\alpha - 1$
  - Common in lattice-based crypto

Why do we care?

Besides a clean abstraction, f-divergences provide a few properties for free:

- **Data-processing inequality:**

$$\mathrm{Div}_f(g(\boldsymbol{P}); g(\boldsymbol{Q})) \leq \mathrm{Div}_f(\boldsymbol{P}; \boldsymbol{Q}) \tag{2}$$

- **Probability-preservation property (PPP):**
  - Consequence of the data-processing inequality
  - Set g so that $g(\boldsymbol{P})$ (resp. $g(\boldsymbol{Q})$) is the outcome (0/1) of a game
- Joint convexity

But in general:

- No tensorisation
- No triangle inequality
- No symmetry

Why not stick to the statistical distance?

The statistical distance Δ_{SD} is a one-size-fits-all divergence, but not always the best:

- The Rényi (R_α) and Kullback-Leibler (D_{KL}) divergences are more tightly connected to {Shannon, collision, min-}entropies
- Hellinger, Kullback-Leibler and χ^2 divergences' tensorisation properties may provide tighter proofs
- The Rényi divergence has a multiplicative probability-preservation property

Rest of this talk: a few selected examples using the Rényi divergence.

Rényi divergence

Let $\boldsymbol{P}, \boldsymbol{Q}$ such that $\operatorname{Supp} \boldsymbol{P} \subseteq \operatorname{Supp} \boldsymbol{Q} = \Omega$. The Rényi divergence of order $\alpha \in [1, \infty]$ between $\boldsymbol{P}$ and $\boldsymbol{Q}$ is defined, for $1 < \alpha < \infty$, as:

$$R_\alpha(\boldsymbol{P}; \boldsymbol{Q}) := \left(\sum_{x \in X} \frac{\boldsymbol{P}(x)^\alpha}{\boldsymbol{Q}(x)^{\alpha-1}} \right)^{\frac{1}{\alpha-1}}, \tag{3}$$

with the two limit cases $R_1(\boldsymbol{P}; \boldsymbol{Q}) = e^{D_{\mathrm{KL}}(\boldsymbol{P};\boldsymbol{Q})}$ and $R_\infty(\boldsymbol{P}; \boldsymbol{Q}) = \max_{x \in \Omega} \frac{\boldsymbol{P}(x)}{\boldsymbol{Q}(x)}$.

Properties: The Rényi divergence is sub-multiplicative (tensorisation) and verifies a weak triangle inequality, but is not symmetric.

Fun fact: the Shannon (H_1), collision (H_2) and min-entropy (H_∞) are part of the class of Rényi entropies H_α, which can be defined as:

$$H_\alpha(X) = \log_2 |\Omega| - \log_2 R_\alpha(\boldsymbol{P}; \boldsymbol{U}), \tag{4}$$

where $X \sim \boldsymbol{P}$, and $\boldsymbol{U}$ is the uniform distribution over Ω.

Probability-preservation properties (PPP)

Let $p = \boldsymbol{P}(E)$ and $q = \boldsymbol{Q}(E)$, so that $0 \leq p, q \leq 1$. This PPP is immediate from (2):

$$\frac{p^\alpha}{q^{\alpha-1}} + \frac{(1-p)^\alpha}{(1-q)^{\alpha-1}} \leq R_\alpha(\boldsymbol{P};\boldsymbol{Q})^{\alpha-1}. \tag{5}$$

(5) admits two special cases:

- An additive PPP [MTMH19]:

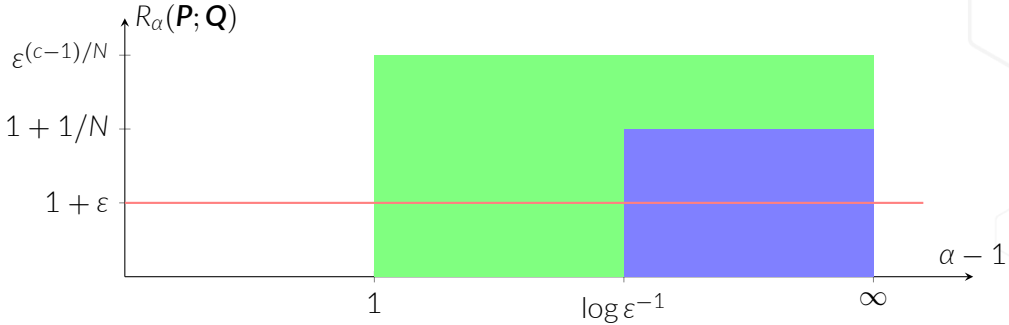
$$|p-q| \leq \sqrt{(R_2(\boldsymbol{P};\boldsymbol{Q})-1)\cdot q} \tag{6}$$

- A multiplicative PPP [LSS14]:

$$p \leq (q \cdot R_\alpha(\boldsymbol{P};\boldsymbol{Q}))^{(\alpha-1)/\alpha}, \tag{7}$$
$$p \leq q \cdot R_\infty(\boldsymbol{P};\boldsymbol{Q}). \tag{8}$$

If N queries to $\boldsymbol{P}$ (resp. $\boldsymbol{Q}$), replace R_α with R_α^N.

Let ε be the advantage when using $\boldsymbol{Q}$ (search: $q = \varepsilon$, decision: $q = 1/2 + \varepsilon$). We want to upper bound the advantage when using $\boldsymbol{P}$. Assume N queries.



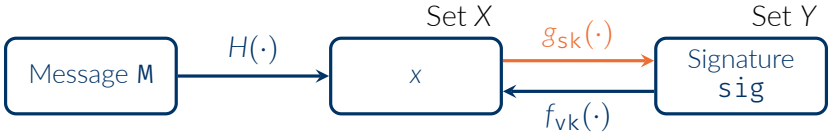
- In green: reductions with polynomial loss (c is a constant > 0)
  - For search problems using (7), see [BLL+15]
  - For decision problems *with the public sampleability property* [BLL+15, MTMH19]
- In blue: search reductions with $O(1)$ loss using (7), see [Pre17]
  - Decision $\Rightarrow$ ad-hoc techniques, e.g. problem switching [NAB+20, DSSS21, LW21]:

$$\text{IND-CPA} \Rightarrow \text{OW-CPA} \Rightarrow \text{IND-CCA} \tag{9}$$

- Below the red line (—): no real advantage over statistical distance

Application 1: Trapdoor Sampling

Most lattice-based H&S constructions [GPV08, MP12, DLP14, PFH+17] rely on trapdoor preimage sampleable functions (TPSFs).



We require that there exists $\boldsymbol{Y}$ of support Y such that for almost all $x \in X$:

$$\{y \leftarrow \boldsymbol{Y} | f_{\mathsf{vk}}(y) = x\} \approx_s \{y \leftarrow g_{\mathsf{sk}}(x)\} \tag{10}$$

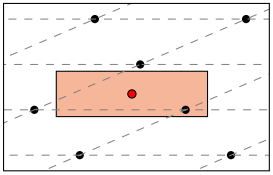
Fun fact/digression: relaxing (10) to hold *on average* over x unlocks more (efficient) constructions [CGM19, DST19, CD20].

Question: Why does $\approx_s$ matter in (10), and how do we assess it?

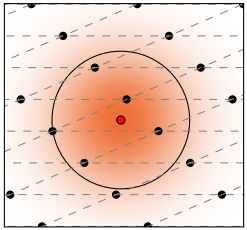
Trapdoor samplers (contained)

Most trapdoor samplers can be seen as randomised variants of (polynomial-time) approximate CVP solvers (e.g. Babai's *nearest plane* and *round-off* algorithms).

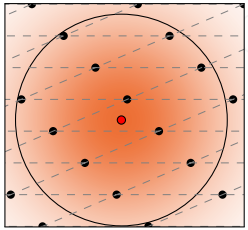
σ too small



The "right" σ

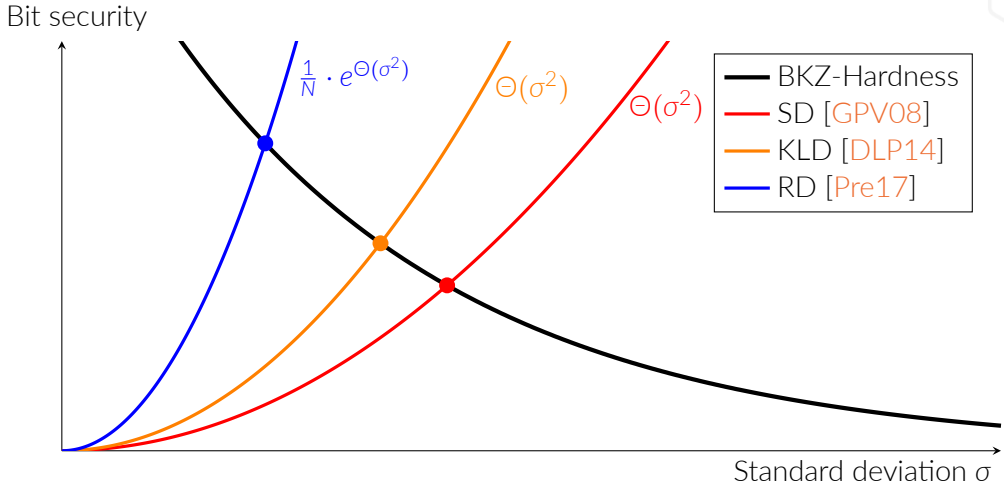


σ too big



1. σ too small $\Rightarrow$ vulnerable to learning attacks [NR06, DN12]
2. σ too large $\Rightarrow$ suboptimal for cryptography

Standard deviation analysis



For Falcon [PFH+17] and $N = 2^{64}$, we gain $\sim$ 30 bits of security (compared to SD).

Application 2: Prime Number Generation in RSA

Prime numbers in RSA

In RSA, the public key is $p \cdot q$.

- Ideally, p, q should be uniform in some high-entropy interval, say $[0, 2^\ell)$
- In practice, most algorithms are very far from providing this guarantee

Can prime number generation be a point of failure in practice? Yes:

1. **Highly structured distributions:** p, q may be recovered by non-generic methods
   - See Coppersmith's attack [Cop96] and its follow-up ROCA [NSS+17]
2. **Insufficient entropy:** distinct public keys may have common factors
   - Compute pairwise GCDs to recover private keys [HDWH12, LHA+12, BCC+13, HFH16, BSTS16, AR18, Kil19]

This is why several prime number generators have been proposed [BD93, Mau95, JPV00, FT14, FT19].

PRIMEINC [BD93]

A simple and popular (PyCrypto, OpenSSL) prime number generator.

PRIMEINC(ℓ, s)

1. Sample odd p uniformly in $\{2^{\ell-1}, \ldots, 2^{\ell}\}$, set $k \leftarrow 0$.
2. While ($k \leq s$) & (p is not prime), set $p \leftarrow p + 2$ and $k \leftarrow k + 1$.
3. If (p is prime) & ($p < 2^{\ell}$), return p, else restart.

Is it secure?

→ **Maybe?**

$$\frac{H_1(PRIMEINC())}{H_1(Uniform)} \underset{\ell \to \infty}{\longrightarrow} 1 \qquad ([BD93])$$

→ **Maybe not?**

$$\Delta_{SD}(PRIMEINC(); Uniform) \underset{\ell \to \infty}{\longrightarrow} 0.86 \qquad ([FT19])$$

Under appropriate conditions (mostly the same as [FT19]):

$$R_\infty(\mathit{PRIMEINC}(); \mathit{Uniform}) = 1 + O(1) \tag{11}$$

Consequence 1: From (8), an RSA-based scheme secure when p, q are sampled uniformly, remains secure (in the single-user setting) when $p, q \leftarrow \mathit{PRIMEINC}()$.

It follows from (11) that:

$$H_2(\mathit{PRIMEINC}()) \geq \ell - O(1) \tag{12}$$

Consequence 2: PRIMEINC is impervious against GCD/common factors attacks.

Conclusion

It is often fruitful to try other divergences than the statistical distance:

- Rényi divergence
  - Lattice-based cryptography [LSS14, BLL+15, Pre17]
  - Differential privacy [Mir17, MTMH19]
  - Prime number generators [AP20]
  - Leakage-resilient cryptography [PGMP19]
- Kullback-Leibler divergence is implicit in the definition of mutual information
- Hellinger distance
  - Key-alternating ciphers [Ste12]
  - Strong randomness extractors [Yas21]
- χ^2 divergence in symmetric cryptography [DHT17]

Not to mention relaxing [DST19, CD20] or strengthening [PGMP19] notions.

Thank You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