Post-Quantum Multi-Recipient KEMs and their Applications

Shuichi Katsumata
AIST, JP

Kris Kwiatkowski
PQShield, UK

Federico Pintore
Univ. of Oxford, UK

Eamonn Postlethwaite
Royal Holloway, UK

Thomas Prest
PQShield, UK/FR

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Main question

How efficiently can we share a session key K between (N + 1) users?

- **Motivation:** Secure group messaging
- **Naive solution with El Gamal:**
  - Send $(g^{r_i}, pk_i^{r_i} \cdot K)$ for each user i
- **Variant by Kurosawa [Kur02]:**
  - Send $(g^r, pk_1^r \cdot K, \ldots, pk_N^r \cdot K)$
  - Asymptotically, saves a factor **2**

Main question

How efficiently can we share a session key K between (N + 1) users?

- **Terminology:** ciphertext compression, mKEM/mPKE, randomness reuse, etc.
- **Two flavors:**
  - Single-message (this work)
  - Multi-message (send a distinct K_i to each user)
- [BBM00, BPS00, Kur02, BBS03, Sma05, HK07, BF07, HTAS09, MH13, Yan15]
- No* post-quantum proposal

This work

- **Revisiting mPKEs & mKEMs**
  - More natural definition
  - Captures classical and post-quantum assumptions
  - QROM security
- **Instantiation from post-quantum assumptions**
  - Lattices
  - Isogenies
  - Efficiency increased by one or two orders of magnitude
- **Application to TreeKEM**
  - Interplay mKEM $\times$ TreeKEM
  - Communication cost divided by 2
- **Focus on lattice-based mKEMs**
  - Concrete security analysis in the mKEM regime
  - New lattice-based mKEMs

Revisiting mPKEs & mKEMs

Full reproducibility [BBS03]:

$$\begin{tikzcd}
\mathrm{Enc}(\mathtt{pk}_1, \mathtt{M}, r) \arrow[rr, "\mathcal{A}(\,\cdot\,, \mathtt{pk}_1, \mathtt{pk}_2, \mathtt{sk}_2, \mathtt{M}')"] & & \mathrm{Enc}(\mathtt{pk}_2, \mathtt{M}', r)
\end{tikzcd}$$

Weak reproducibility [BF05]:

$$\begin{tikzcd}
\mathrm{Enc}(\mathtt{pk}_1, \mathtt{M}, r) \arrow[rr, "\mathcal{A}(\,\cdot\,, \mathtt{pk}_1, \mathtt{pk}_2, \mathtt{sk}_2, \mathtt{M})"] \arrow[dr, "\mathcal{A}(\,\cdot\,, \mathtt{pk}_2, \mathtt{pk}_3, \mathtt{sk}_3, \mathtt{M}')"'] & & \mathrm{Enc}(\mathtt{pk}_2, \mathtt{M}, r) \arrow[dl, "\mathcal{A}(\,\cdot\,, \mathtt{pk}_1, \mathtt{pk}_3, \mathtt{sk}_3, \mathtt{M}')"] \\
& * &
\end{tikzcd}$$

Decomposability (this work):

$$\mathrm{Enc}(\mathtt{pk}_i, \mathtt{M}) = \overbrace{\mathtt{ct}_0}^{\mathrm{Enc}^{\mathtt{ind}}(r_0)} \; \overbrace{\widehat{\mathtt{ct}}_i}^{\mathrm{Enc}^{\mathtt{dep}}(\mathtt{pk}_i, \mathtt{M}, r_0, r_i)}$$

	Fully rep.	Weakly rep.	Decomposable
El Gamal	✓	✓	✓
LP [LP11]	?	?	✓
SIDH [JD11, DJP14]	?	?	✓
CSIDH	?	?	✓

Example: El Gamal. Let a ciphertext $\mathtt{ct} = (g^r, \mathsf{pk}_1^r \cdot \mathtt{M})$ with $\mathsf{pk}_1 = g^{\mathsf{sk}_1}$.

- **Full reproducibility:** $(g^r, *) \longrightarrow \left(g^r, (g^r)^{\mathsf{sk}_2} \cdot \mathtt{M}'\right)$.
- **Decomposability:** $(\mathtt{ct}_0 = g^r, \widehat{\mathtt{ct}}_1 = \mathsf{pk}_1^r \cdot \mathtt{M})$.

A ciphertext with N recipients will be $\vec{\mathtt{ct}} = (\mathtt{ct}_0, \widehat{\mathtt{ct}}_1, \ldots, \widehat{\mathtt{ct}}_N)$.
Key generation and decryption remain the same.

Generic Transforms

[BBS03]	**Fully rep.** IND-CPA PKE	➡	**Multi**-msg IND-**CPA** mPKE	SM
[BBS03]	**Fully rep.** IND-**CCA** PKE	➡	**Multi**-msg IND-CCA mPKE	SM
[BF07]	**Weakly rep.** IND-CPA PKE	➡	**Single**-msg IND-CCA mPKE	ROM
Our work	**Decomposable single**-msg IND-CPA **m**PKE	➡	**Single**-msg IND-CCA mKEM	QROM

Generic Transforms

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Our work	**Decomposable single**-msg IND-CPA **m**PKE	➜	**Single**-msg IND-CCA mKEM	QROM

Open question: Can we (dis)prove the following statement?

(Decomposable **multi**-msg IND-CPA mPKE) ➜ (**Multi**-msg IND-CCA mKEM)

Decomp. IND-CPA mPKE $\Rightarrow$ IND-CCA mKEM

Encaps$(\{\mathsf{pk}_1, \ldots, \mathsf{pk}_N\})$

1. Generate a random M
2. $\mathsf{ct}_0 \leftarrow \mathsf{Enc}^{\mathsf{ind}}(G_1(\mathsf{M}))$
3. For $i = 1, \ldots, N$:
   - $\widehat{\mathsf{ct}}_i \leftarrow \mathsf{Enc}^{\mathsf{dep}}(\mathsf{pk}_i, \mathsf{M}, G_1(\mathsf{M}), G_2(\mathsf{pk}_i, \mathsf{M}))$
4. $\mathsf{K} := H(\mathsf{M})$
5. Return $(\mathsf{K}, \vec{\mathsf{ct}} := (\mathsf{ct}_0, (\widehat{\mathsf{ct}}_i)_{i \in [N]}))$

Decaps$(\mathsf{pk}_i, \mathsf{ct} = (\mathsf{ct}_0, \widehat{\mathsf{ct}}_i))$

1. $\mathsf{M} \leftarrow \mathsf{Dec}(\mathsf{sk}_i, \mathsf{ct})$
2. If $M = \perp$, return $\mathsf{K} := \perp$
3. $\mathsf{ct}_0 \leftarrow \mathsf{Enc}^{\mathsf{ind}}(G_1(\mathsf{M}))$
4. $\widehat{\mathsf{ct}}_i \leftarrow \mathsf{Enc}^{\mathsf{dep}}(\mathsf{pk}_i, \mathsf{M}, G_1(\mathsf{M}), G_2(\mathsf{pk}_i, \mathsf{M}))$
5. If $(\mathsf{ct}_0, \widehat{\mathsf{ct}}_i) \neq \mathsf{ct}$, return $\mathsf{K} := \perp$
6. Return $\mathsf{K} = H(\mathsf{M})$

- G_1, G_2 are PRFs, H is a hash function, all are modeled as random oracles.
- QROM proof uses compressed oracles [Zha19].
- We can achieve implicit rejection as well.

Instantiation from Post-Quantum Assumptions

The Lindner–Peikert framework [LP11]

Keygen ($\mathbf{A} \in \mathcal{R}_q^{m\times m}$)

1. Sample short matrices $\mathbf{S}, \mathbf{E}$
2. $\mathbf{B} \leftarrow \mathbf{AS} + \mathbf{E}$
3. $\mathsf{sk} := (\mathbf{S}, \mathbf{E}), \mathsf{pk} := \mathbf{B}$

Enc$(\mathsf{pk}, \mathtt{M})$

1. Sample short matrices $\mathbf{R}, \mathbf{E}', \mathbf{E}''$
2. $\mathbf{U} \leftarrow \mathbf{RA} + \mathbf{E}'$
3. $\mathbf{V} \leftarrow \mathbf{RB} + \mathbf{E}'' + \mathrm{Encode}(\mathtt{M})$
4. $\mathtt{ct} := (\mathbf{U}, \mathbf{V})$

Dec$(\mathsf{sk}, \mathtt{ct})$

1. $\mathtt{M} \leftarrow \mathrm{Decode}(\mathbf{V} - \mathbf{US})$

Encompasses these NIST Round 3 candidates:

- FrodoKEM
- Kyber
- NTRU LPRime
- Saber

The Lindner-Peikert framework is decomposable:

- Use the same $\mathbf{A}$ for all public keys.
- $\mathbf{U}$ is then independent of $\mathtt{pk}$ and $\mathtt{M}$.

$\mathsf{Enc}(\mathsf{pk} = \mathbf{B}, \mathtt{M})$

1. Sample short matrices $\mathbf{R}, \mathbf{E}', \mathbf{E}''$
2. $\mathbf{U} \leftarrow \mathbf{R}\mathbf{A} + \mathbf{E}'$
3. $\mathbf{V} \leftarrow \mathbf{R}\mathbf{B} + \mathbf{E}'' + \mathsf{Encode}(\mathtt{M})$
4. $\mathtt{ct} := (\mathbf{U}, \mathbf{V})$

The Lindner-Peikert framework is decomposable:

- → Use the same $\mathbf{A}$ for all public keys.
- → $\mathbf{U}$ is then independent of pk and $\mathtt{M}$.

$\mathsf{Enc}(\mathsf{pk} = \mathbf{B}, \mathtt{M})$

1. Sample short matrices $\mathbf{R}, \mathbf{E}', \mathbf{E}''$
2. $\mathbf{U} \leftarrow \mathbf{RA} + \mathbf{E}'$
3. $\mathbf{V} \leftarrow \mathbf{RB} + \mathbf{E}'' + \mathsf{Encode}(\mathtt{M})$
4. $\mathtt{ct} := (\mathbf{U}, \mathbf{V})$

$\Longrightarrow$

$\mathsf{MultiEnc}(\{\mathsf{pk}_1, \ldots, \mathsf{pk}_N\}, \mathtt{M})$

1. Sample short matrices $\mathbf{R}, \mathbf{E}'$
2. $\mathbf{U} \leftarrow \mathbf{RA} + \mathbf{E}'$
3. For $i = 1, \ldots, N$:
   1. Sample a short matrix $\mathbf{E}''_i$
   2. $\mathbf{V}_i \leftarrow \mathbf{RB}_i + \mathbf{E}''_i + \mathsf{Encode}(\mathtt{M})$
4. $\vec{\mathtt{ct}} := (\mathbf{U}, \mathbf{V}_1, \ldots, \mathbf{V}_N)$

Each $\mathbf{V}_i$ is much smaller and faster to compute than $\mathbf{U}$:

- → Shorter dimensions
- → Bit dropping

Security reduces to LWE with many samples (see end of the talk).

SIDH [JD11, DJP14] and SIKE

- E is an elliptic curve
- $E[\ell_A^a] = \langle P_A, Q_A \rangle$
- $E[\ell_B^b] = \langle P_B, Q_B \rangle$

Keygen(E, P_A, Q_A, P_B, Q_B)

1. $\mathtt{sk} := \psi$, where $\psi : E \to E/\langle R_B \rangle$ is an isogeny of kernel R_B
2. $\mathtt{pk} := (E/\langle R_B \rangle, \psi(P_A), \psi(Q_A))$

Enc$(\mathtt{pk}, \mathtt{M})$

1. Sample an isogeny $\varphi : E \to E/\langle R_A \rangle$
2. $\mathtt{ct}_0 = (E/\langle R_A \rangle, \varphi(P_B), \varphi(Q_B))$
3. Compute $j = \text{j-Inv}(E/\langle R_A, R_B \rangle)$
4. $\widehat{\mathtt{ct}} = j \oplus \mathtt{M}$
5. $\mathtt{ct} := (\mathtt{ct}_0, \widehat{\mathtt{ct}})$

Dec$(\mathtt{sk}, \mathtt{ct})$

1. Compute $j = \text{j-Inv}(E/\langle R_A, R_B \rangle)$
2. $\mathtt{M} = j \oplus \widehat{\mathtt{ct}}$

SIDH [JD11, DJP14] and SIKE

- E is an elliptic curve
- $E[\ell_A^a] = \langle P_A, Q_A \rangle$
- $E[\ell_B^b] = \langle P_B, Q_B \rangle$

Keygen(E, P_A, Q_A, P_B, Q_B)

1. $\mathtt{sk}_i := \psi_i$, where $\psi_i : E \to E/\langle R_B \rangle$ is an isogeny of kernel $R_B^{(i)}$
2. $\mathtt{pk} := (E/\langle R_B^{(i)} \rangle, \psi_i(P_A), \psi_i(Q_A))$

Security reduces to SSDDH [DJP14].

Enc$(\{\mathtt{pk}_1, \ldots, \mathtt{pk}_N\}, \mathtt{M})$

1. Sample an isogeny $\varphi : E \to E/\langle R_A \rangle$
2. $\mathtt{ct}_0 = (E/\langle R_A \rangle, \varphi(P_B), \varphi(Q_B))$
3. For $i = 1, \ldots, N$:
   1. Compute $j_i = \text{j-Inv}(E/\langle R_A, R_B^{(i)} \rangle)$
   2. $\widehat{\mathtt{ct}}_i = j_i \oplus \mathtt{M}$
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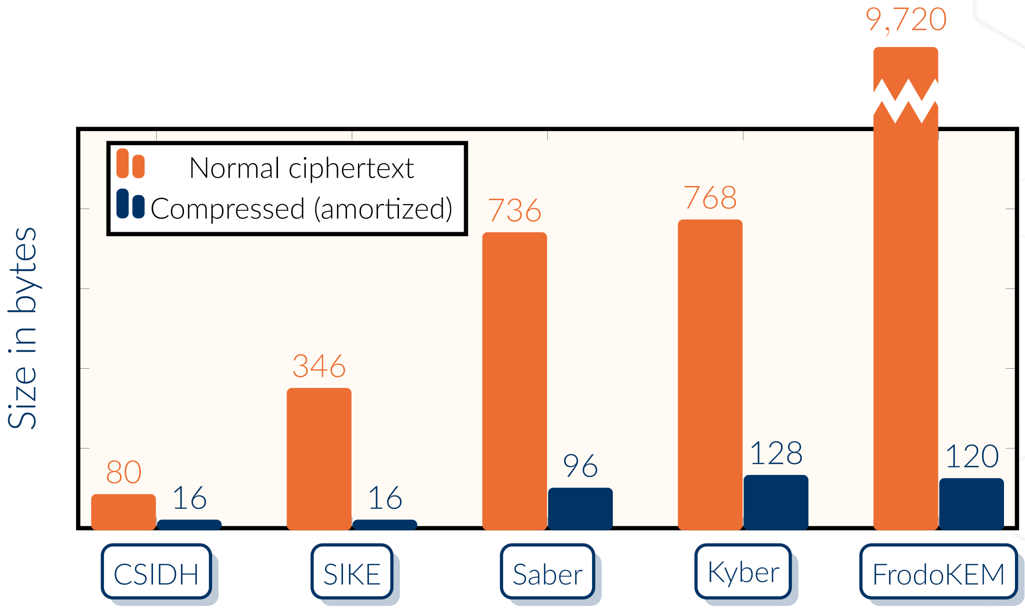
Dec$(\mathtt{sk}_i, (\mathtt{ct}_0, \widehat{\mathtt{ct}}_i))$

1. Compute $j_i = \text{j-Inv}(E/\langle R_A, R_B^{(i)} \rangle)$
2. $\mathtt{M} = j_i \oplus \widehat{\mathtt{ct}}_i$

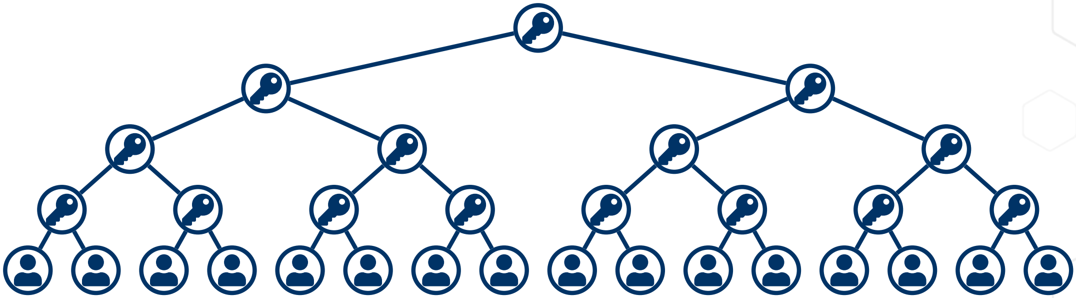
Impact on 1 PKE + 4 KEMs (NIST level I)



Application to TreeKEM

TreeKEM [BBR18, BBM+20, OBR+20, ACDT20]:

- → Key component of the MLS draft IETF proposal for group messaging
- → The N users are arranged as leaves of a (binary) tree
- → **TreeKEM invariant:** 👤 knows a private 🔑 if and only if it is in its path.



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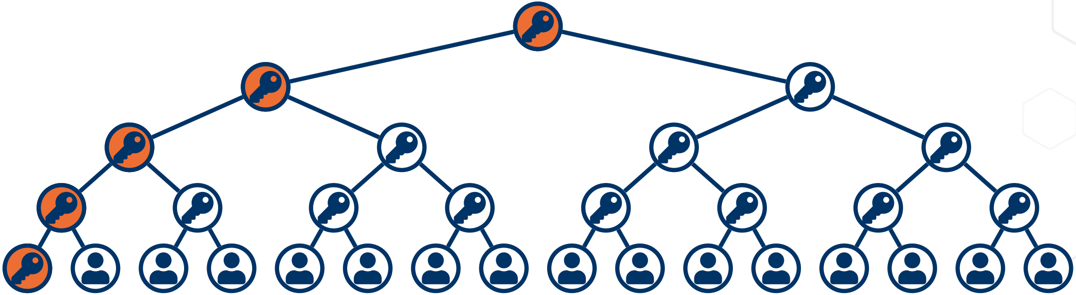

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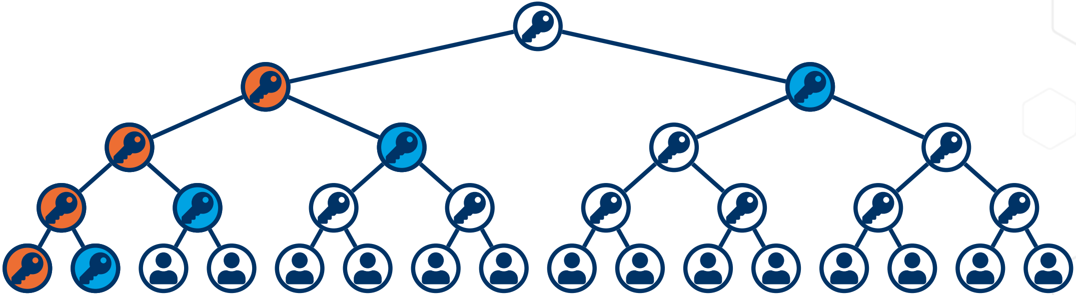
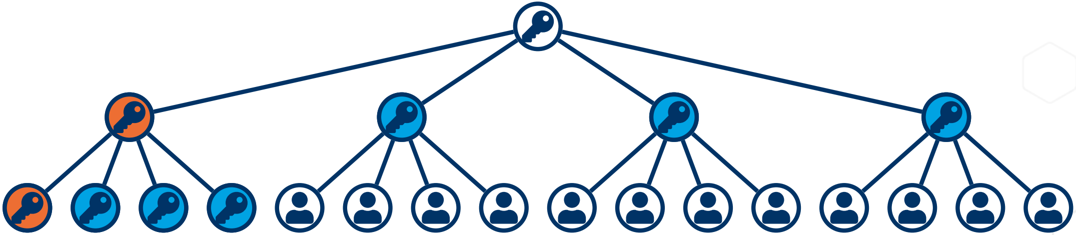


Post-compromise security: Users refresh their key material by broadcasting an update package that contains:

- One `pk` for each node in the path (except the root).
- One `ct` for each node in the co-path (siblings of nodes in the path).

What if we use a m-ary tree instead of a binary tree?

- We send $\log_m(N)$ public keys and $(m-1) \cdot \log_m(N)$ ciphertexts
- However all ciphertexts at a same level encapsulate the same key!
- We can use a single mKEM ciphertext at each level



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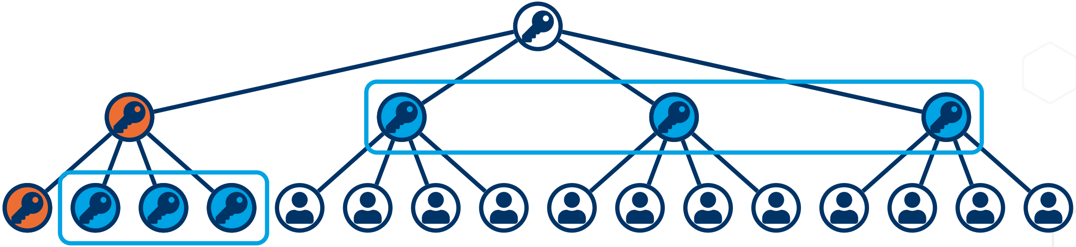

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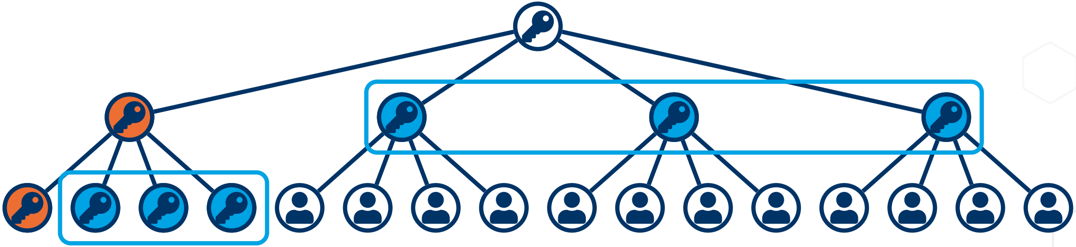
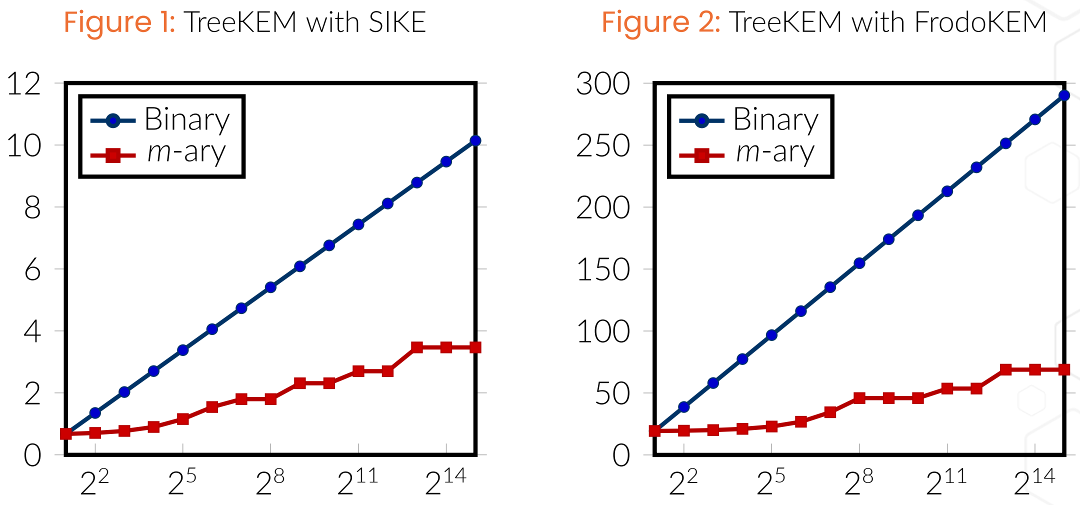


Size of an update package:

- **Standard TreeKEM:** $\log_2(N)\cdot(|\mathsf{pk}|+|\mathsf{ct}_0|+|\widehat{\mathsf{ct}}_i|)$
- **m-ary trees + mKEM:** $\log_m(N)\cdot(|\mathsf{pk}|+|\mathsf{ct}_0|+|\widehat{\mathsf{ct}}_i|\cdot m)$

Size of an update package in kilobytes as a function of number of users (NIST level I)



Note for later: for all examples, the package size is $\propto \log N$.

Focus on Lattice-Based mKEMs

We study the concrete security of the mKEMs based on:

- Kyber
- Saber
- FrodoKEM

We also propose two (more) efficient lattice-based mKEMs:

- **BilboKEM** (based on FrodoKEM)
- **Ilum** (based on Kyber)

All schemes in this slide instantiate the Lindner-Peikert framework.

- Security for key recovery: LWE/LWR with finite samples
- Security for encryption: LWE/LWR with **unbounded** samples

⚠ Work in progress

How is it related to LWE?

MultiEnc$(\{\mathsf{pk}_1, \ldots, \mathsf{pk}_N\}, \mathtt{M})$

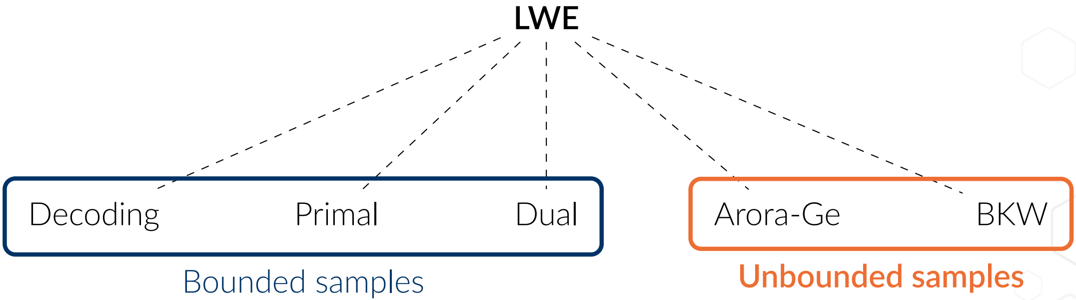
1. Sample short matrices $\mathbf{R}, \mathbf{E}', \{\mathbf{E}''_1, \ldots, \mathbf{E}''_N\}$
2. $\mathbf{U} \leftarrow \mathbf{R}\mathbf{A} + \mathbf{E}'$
3. For $i = 1, \ldots, N$:
   1. $\mathbf{V}_i \leftarrow \mathbf{R}\mathbf{B}_i + \mathbf{E}''_i + \mathrm{Encode}(\mathtt{M})$
4. $\vec{\mathtt{ct}} := (\mathbf{U}, \mathbf{V}_1, \ldots, \mathbf{V}_N)$

Attacker's goal: Given $(\mathbf{U}, \mathbf{V}_1, \ldots, \mathbf{V}_N)$, recover $\mathtt{M}$.

Note that:

$$(\mathbf{U} \| \mathbf{V}_1 \| \ldots \| \mathbf{V}_N) = \mathbf{R} \times (\mathbf{A} \| \mathbf{B}_1 \| \ldots \| \mathbf{B}_N) + (\mathbf{E}' \| \mathbf{E}''_1 \| \ldots \| \mathbf{E}''_N) + (\mathbf{0} \| \mathtt{M} \| \ldots \| \mathtt{M})$$

Attacks Against LWE (Very Contained)



Methodology

Primal attack:

- Leaky-LWE framework: `https://github.com/lducas/leaky-LWE-Estimator/tree/NIST-round3`
- Determining the block Size: we use the PIM (as opposed to GSA)
- Fitting function for "dimensions for free": same as Kyber/FrodoKEM

Arora-Ge:

- LWE estimator: `https://lwe-estimator.readthedocs.io/en/latest/_apidoc/estimator/estimator.arora_gb.html`
- Take rounding into account

BKW:

- Based on Coded-BKW
- Additional rounding **not** taken into account
- Next step: [BGJ+20] and quantum

Attacks with Unbounded Samples

Arora-Ge:

- Algebraic attack (linearization)
- Requires $n^{O(d)}$ samples, where $d = \max(\|\mathbf{E}'\|_\infty, \|\mathbf{E}_i''\|_\infty)$

BKW:

- Combinatorial attack (lattice reduction + guessing)
- Complexity: $2^{\tilde{O}(n)}$

Scheme	$\|\mathbf{E}'\|_\infty$	$\|\mathbf{E}_i''\|_\infty$
Kyber	≥ 5	≥ 100
Saber	$= 8$	≥ 16
FrodoKEM	≥ 13	≥ 13
NTRU LPRime	$= 3$	≥ 288

In effect, the $\|\mathbf{E}_i''\|_\infty$ are large thanks to bit dropping in the $\mathbf{V}_i$.

- Rules out Arora-Ge and BKW in practice!

Security Metrics at NIST Level 1 ($\approx$ AES-128)

S = Samples, C = Classical, Q = Quantum, G = Gates, O = Operations.

	Primal			Arora-Ge			BKW		
Scheme	S.	CG	QG	S	CO	QO	S	CO	QO
Saber	512	152	142	∞	2646	TBD	∞	158	TBD
Kyber	512	151	143	∞	2408	TBD	∞	147	TBD
FrodoKEM	656	175	164	∞	2798	TBD	∞	226	TBD

Same trends at high security levels.

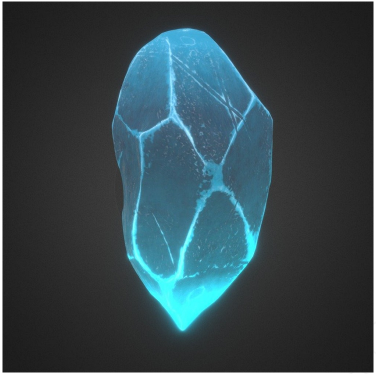
BilboKEM and Ilum

We propose lattice-based mKEMS that are tailored to mKEM constraints:

- **"Standard":** Minimize $|\widehat{\mathtt{ct}}_i|$
- **TreeKEM:** Minimize $\tau = \min_m \frac{|\mathtt{pk}|+|\mathtt{ct}_0|+|\widehat{\mathtt{ct}}_i|\cdot m}{\log_2(m)}$

BilboKEM (variant of FrodoKEM)

Ilum (variant of Kyber)



How do we minimize the bitsize of the V_i?

Recall that $\mathtt{M} = \mathrm{Decode}(\mathbf{V}_i - \mathtt{sk}_i \cdot \mathbf{U})$.

We leverage the following tools:

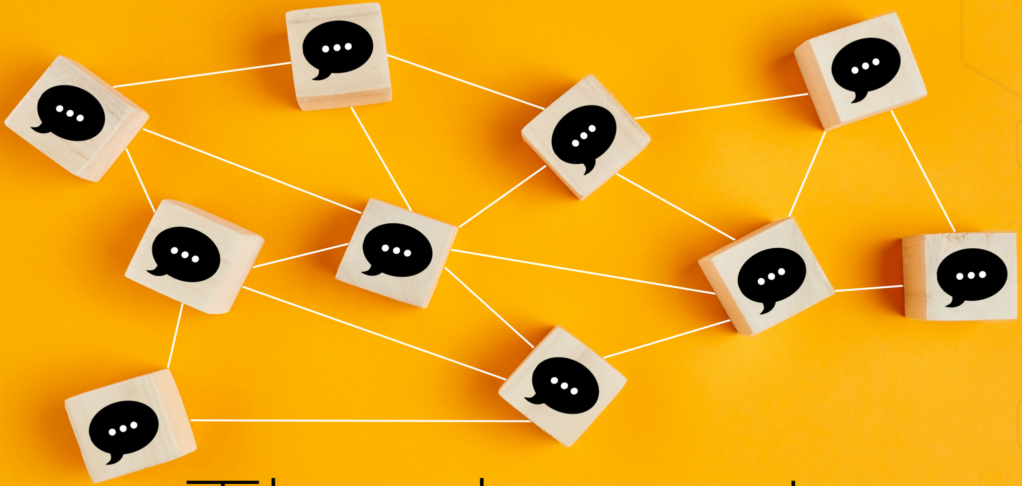
- **Bit dropping:** drop the least significant bits of $\mathbf{V}_i$
  - Reduces the size of $\mathbf{V}_i$, increase the LWE error rate
  - Increases the decryption failure rate
- **Coefficent dropping:** drop superflous coefficients of $\mathbf{V}$
  - Reduces the size of $\mathbf{V}_i$
  - None!
- **Increase the modulus q**
  - Allows to pack more bits per coefficient of $\mathbf{V}_i$
  - Increases the size of $\mathbf{U}$, decreases the LWE error rate
- **Error correcting codes** (we discarded this option)
  - Decreases the decryption failures rate
  - Timing attacks, delicate security analysis [DVV19, GJY19, DTVV19]

BilboKEM

	FrodoKEM-640	BilboKEM-640
Dimension n	640	640
Dimensions $\bar{m} \times \bar{n}$	8×8	8×8
Modulus q	2^{15}	$\mathbf{2^{16}}$
Std. dev. σ	2.8	2.8
Bits dropped in $\mathbf{V}_i$	0 out of 15	**13** out of 16
(Key bits) / coef.	2 out of 15	2 out of 3
Lattice (S/CG/QG)	656/152/142	∞/162/152
Arora-Ge (S/CO/QO)	∞/2798/TBD	∞/4601/TBD
BKW (S/CO/QO)	∞/226/TBD	∞/224/TBD
Decryption failure rate	2^{-138}	2^{-129}
$\lvert$pk$\rvert$ (bytes)	9600	10240
$\lvert\mathbf{U}\rvert$ (bytes)	9600	10240
$\lvert\mathbf{V}_i\rvert$ (bytes)	120	**24**

Ilum

	Kyber-512	Ilum-512-C
Ring degree n	256	256
Module rank ℓ	2	2
Modulus q	3329	3329
Error param. (η_1, η_2)	(3, 2)	(3, 2)
Bits / coef. in $(\mathbf{U}, \mathbf{V}_i)$	(10, 4)	**(11, 3)**
Coef. dropped in $\mathbf{V}_i$	0 out of 256	**128** out of 256
(Key bits) / coef.	1 out of 4	1 out of 3
Lattice (S/CG/QG)	512/151/143	∞/150/142
Arora-Ge (S/CO/QO)	∞/2408/TBD	∞/2227/TBD
BKW (S/CO/QO)	∞/147/TBD	∞/157/TBD
Decryption failure rate	2^{-139}	2^{-125}
$\lvert$pk$\rvert$ (bytes)	768	**768**
$\lvert\mathbf{U}\rvert$ (bytes)	640	**704**
$\lvert\mathbf{V}_i\rvert$ (bytes)	128	**48**



Thank you!

https://eprint.iacr.org/2020/1107

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