Basic Constructions over Lattices I: Key Establishment

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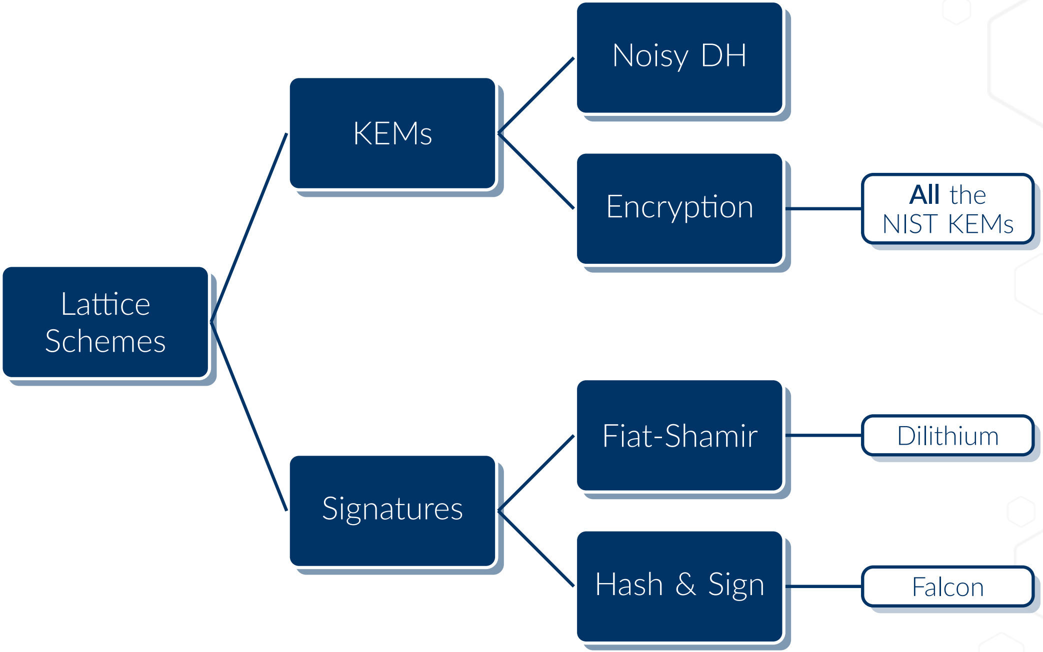
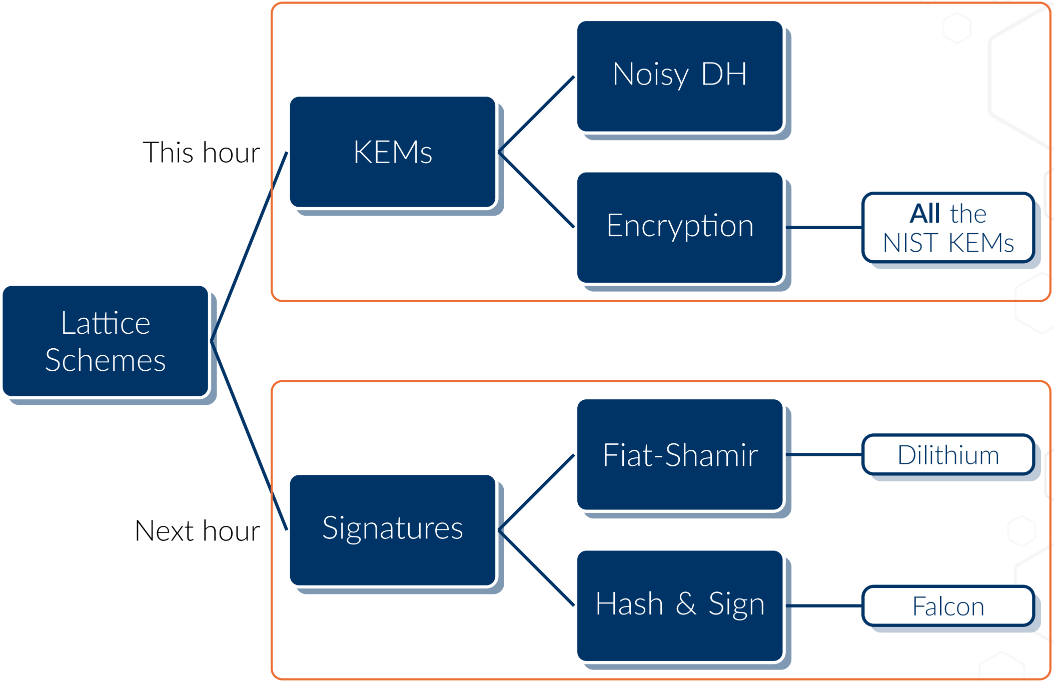
Introduction


Lattice Schemes
KEMs
Noisy DH
Encryption
All the NIST KEMs
Signatures
Fiat-Shamir
Dilithium
Hash & Sign
Falcon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Roadmap

Approach of this course:

- Proceed by analogy between number-theoretic vs lattice-based schemes (e.g. El Gamal vs "noisy El Gamal" [LPR10, LP11])
- This way, we can separate paradigm (e.g. Fiat-Shamir) and assumption (e.g. LWE)
  - Allows to understand which notions are intrinsic to the paradigm or are assumption-dependent
  - Straightforward adaptations can fail, and it is important to understand why
  - In general, it is useful to understand where two assumptions (e.g. LWE vs DLOG) may be similar, and where they differ

Plan:

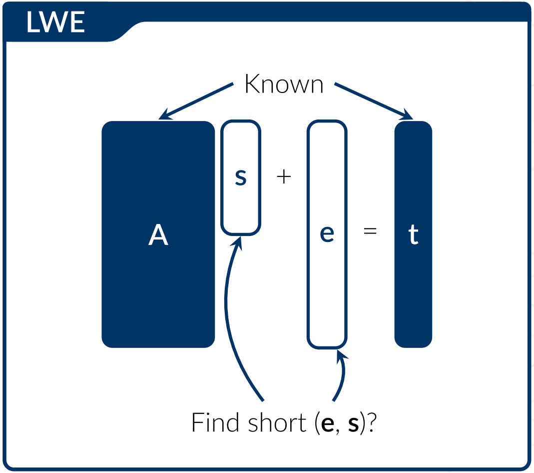
- *This hour:* key-establishment schemes based on LWE
  - There exist encryption based on NTRU as well.
    We ignore them here, but most of our remarks apply.
- *Next hour:* signature schemes based on LWE/SIS/NTRU

Learning with Errors (LWE)

The LWE assumption:

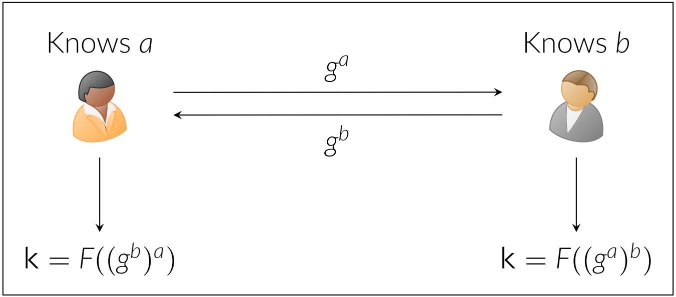
- Two fundamental variants:
  - *Search*: hard to find $(\mathbf{s}, \mathbf{e})$
  - *Decision*: hard to distinguish $(\mathbf{A}, \mathbf{t})$ from uniformly random $(\mathbf{A}, \mathbf{u})$
- Very versatile:
  - Base ring $\mathcal{R}$ $(\mathbb{Z}, \mathbb{Z}[x]/(x^d + 1), \dots)$
  - Dimensions of the vectors/matrices
  - Distributions (binomial, uniform, etc.)
- Resilient to some extent to bit dropping
  - If we drop the least significant bits of $\mathbf{t}$, $(\mathbf{A}, \mathbf{t})$ is still useful.
  - See also LWR, where $\mathbf{t} = \mathrm{MSB}(\mathbf{A} \cdot \mathbf{s})$.
- We will liken a lot LWE to DLOG:
  - *LWE*: $(\mathbf{A}, \mathbf{t} = \mathbf{A}\mathbf{s} + \mathbf{e}) \xrightarrow{\mathcal{A}} (\mathbf{s}, \mathbf{e})$?
  - *DLOG*: $(g, h = g^x) \xrightarrow{\mathcal{A}} x$?

  This analogy is convenient when it holds, and informative when it fails.



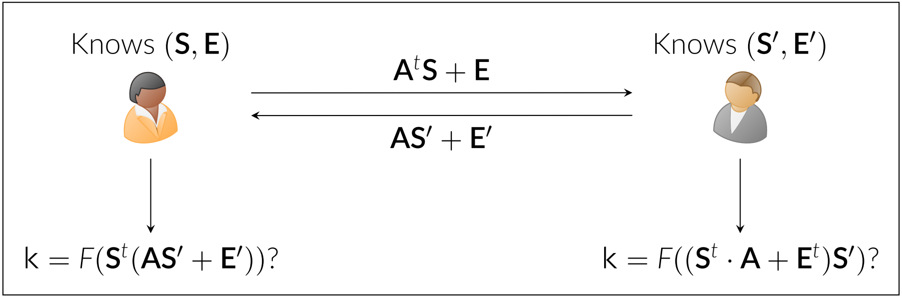
Noisy Diffie-Hellman

Diffie-Hellman



A few comments:

- It works thanks to the commutativity of exponentiation: $(g^a)^b = (g^b)^a = g^{ab}$
- Nice properties: fully non-interactive, can be used with static keys, etc.



- Parties have to "pick a side", but that isn't the big issue.
- Bigger problem: and agree on a shared secret *up to some additive noise*:

$$\mathbf{S}^t(\mathbf{A}\mathbf{S}' + \mathbf{E}') = \mathbf{S}^t\mathbf{A}\mathbf{S}' + \underline{\mathbf{S}^t\mathbf{E}'} \approx \mathbf{S}^t\mathbf{A}\mathbf{S}' + \underline{\mathbf{E}^t\mathbf{S}'} = (\mathbf{S}^t \cdot \mathbf{A} + \mathbf{E}^t)\mathbf{S}' \qquad (1)$$

- Natural idea: keep most significant bits (MSB) of $\mathbf{S}^t\mathbf{A}\mathbf{S}' + \underline{\mathbf{S}^t\mathbf{E}'}$ and $\mathbf{S}^t\mathbf{A}\mathbf{S}' + \underline{\mathbf{E}^t\mathbf{S}'}$.
  - But does it work?

Why keeping the MSBs doesn't work

For this example, assume $q = 2^K$. Suppose that at a given coefficient:

1. $\mathbf{S}^t\mathbf{A}\mathbf{S}'$ is extremely close to $q/2$ or 0
2. $\mathbf{S}^t\mathbf{E}' > 0$
3. $\mathbf{E}^t\mathbf{S}' < 0$

Then  and  might not agree on the MSB of that coefficient.

A few comments:

- →  and  have no way to detect that they disagreed on a coefficient.
- → Discarding coefficients that are "too close to $q/2$" doesn't work.
- → One naive workaround is to set $q = \Omega(2^\lambda)$, but this raises its own problems.

The results of [GKRS20] suggest that if $q = o(2^\lambda)$, then finding a function F such that

$$F(\mathbf{S}^t(\mathbf{A}\mathbf{S}' + \mathbf{E}')) = F((\mathbf{S}^t \cdot \mathbf{A} + \mathbf{E}^t)\mathbf{S}') \tag{2}$$

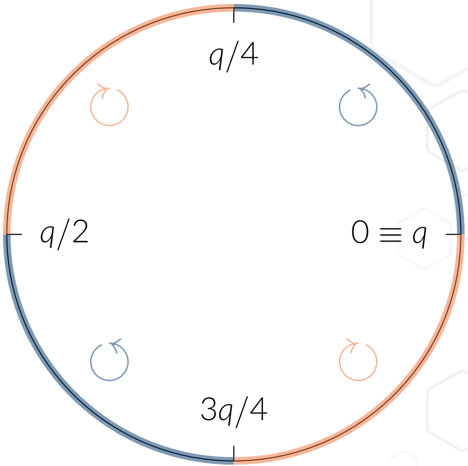
is a difficult problem.

The solution: reconciliation [DXL12, Pei14]

For each coefficient x, sends 1 additional "reconciliation bit" $\texttt{rec}(x)$: ↺ / ↻

- If we identify $\mathbb{Z}_q$ with a circle, ↺ / ↻ indicates in which direction rounded the coefficient.
- This allows and to agree on "*borderline*" coefficients. For example, if for a given x:
  1. has $0.501 \cdot q$
  2. has $0.499 \cdot q$

  Then sends ↺ and rounds to 1 (instead of 0 if there was no reconciliation bit).



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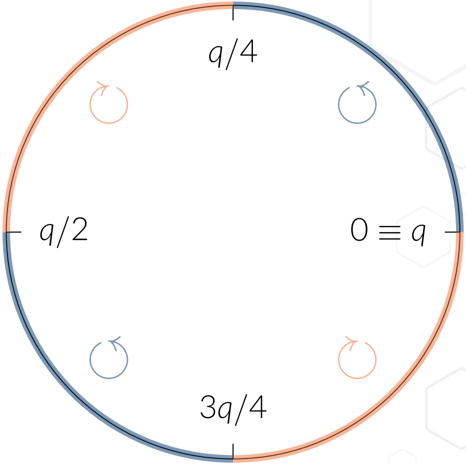

Question: Given $q = 2^\kappa$ and $x \leftarrow \mathbb{Z}_q$, does $\mathtt{rec}(x)$ leak information about MSB(x)?

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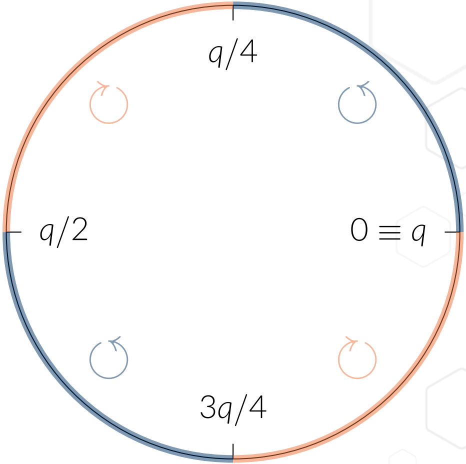
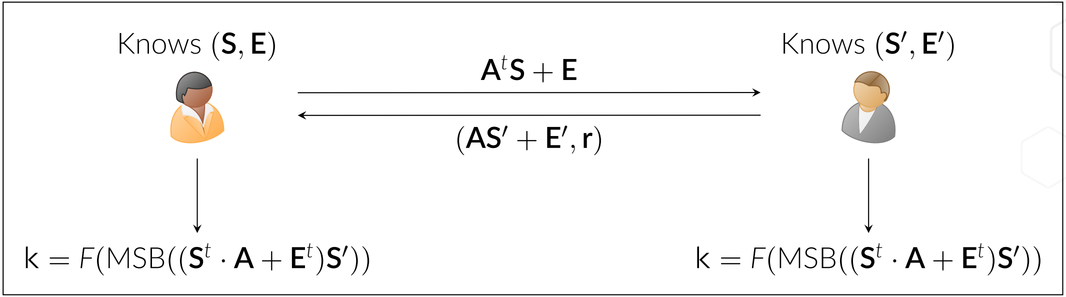


Question: Given $q = 2^\kappa$ and $x \leftarrow \mathbb{Z}_q$, does $\texttt{rec}(x)$ leak information about $\mathrm{MSB}(x)$?
Answer: No. Computation below for the case $(\texttt{rec}(x) = \circlearrowleft)$:

$$\mathbb{P}_{x \leftarrow \mathbb{Z}_p}[\mathrm{MSB}(x) = 0 \mid \texttt{rec}(x) = \circlearrowleft] = \frac{\mathbb{P}_{x \leftarrow \mathbb{Z}_p}[(\mathrm{MSB}(x) = 0) \wedge (\texttt{rec}(x) = \circlearrowleft)]}{\mathbb{P}_{x \leftarrow \mathbb{Z}_p}[\texttt{rec}(x) = \circlearrowleft]} = \frac{q/4}{q/2} = \frac{1}{2}$$

Noisy Diffie-Hellman: now with reconciliation



Variations on the reconciliation idea:

- → More coefficients than bits of k: use error-correcting codes [ADPS16]
- → Less coefficients than bits of k: encode several bits per coefficient [BCD+16]
- → Simply discard coefficients that are borderline for  [Saa17]

Noisy Diffie-Hellman: now with reconciliation

Knows $(\mathbf{S}, \mathbf{E})$

$$\xrightarrow{\mathbf{A}^t\mathbf{S} + \mathbf{E}}$$

$$\xleftarrow[(\mathbf{A}\mathbf{S}' + \mathbf{E}', \mathbf{r})]{}$$

Knows $(\mathbf{S}', \mathbf{E}')$

$$\mathrm{k} = F(\mathrm{MSB}((\mathbf{S}^t \cdot \mathbf{A} + \mathbf{E}^t)\mathbf{S}'))$$

$$\mathrm{k} = F(\mathrm{MSB}((\mathbf{S}^t \cdot \mathbf{A} + \mathbf{E}^t)\mathbf{S}'))$$

Some fundamental differences with classical Diffie-Hellman:

- → Reconciliation makes the protocol interactive
- → Cannot be used with static keys due to an attack by Fluhrer [Flu16]

Noisy El Gamal

El Gamal

Keygen$(g \in \mathbb{G})$

1. Sample $x \leftarrow \mathbb{Z}_{|\mathbb{G}|}$
2. $h \leftarrow g^x$
3. $\mathsf{dk} := x, \mathsf{ek} := h$

Enc$(\mathbf{msg}, \mathbf{ek})$

1. Sample $r \leftarrow \mathbb{Z}_{|\mathbb{G}|}$
2. $u \leftarrow g^r$
3. $v \leftarrow h^r \cdot \mathtt{msg}$
4. $\mathsf{c} := (u, v)$

Dec$(\mathbf{c}, \mathbf{dk})$

1. $\mathtt{msg} \leftarrow v \cdot u^{-x}$

"Noisy" El Gamal [LPR10, LP11]

Keygen$(\mathbf{A} \in \mathcal{R}_q^{m \times m})$

1. Sample short $\mathbf{S}, \mathbf{E}$
2. $\mathbf{B} \leftarrow \mathbf{AS} + \mathbf{E}$
3. $\mathsf{dk} := (\mathbf{S}, \mathbf{E}), \mathsf{ek} := \mathbf{B}$

Enc$(\mathbf{msg}, \mathbf{ek})$

1. Sample short $\mathbf{R}, \mathbf{E}', \mathbf{E}''$
2. $\mathbf{U} \leftarrow \mathbf{RA} + \mathbf{E}'$
3. $\mathbf{V} \leftarrow \mathbf{RB} + \mathbf{E}'' + \mathtt{Encode}(\mathtt{msg})$
4. $\mathsf{c} := (\mathbf{U}, \mathbf{V})$

Dec$(\mathbf{c}, \mathbf{dk})$

1. $\mathtt{msg} \leftarrow \mathtt{Decode}(\mathbf{V} - \mathbf{US})$

Correctness

Decryption is successful since:

- **El Gamal:** $v \cdot u^{-x} = (h^r \cdot \mathtt{msg}) \cdot (g^r)^{-x} = \mathtt{msg}$
- **Noisy El Gamal:**

$$\begin{aligned} \mathbf{V} - \mathbf{U}\mathbf{S} &= \big(\mathbf{R}(\mathbf{A}\mathbf{S} + \mathbf{E}) + \mathbf{E}'' + \mathtt{Encode}(\mathtt{msg})\big) - (\mathbf{R}\mathbf{A} + \mathbf{E}')\mathbf{S} && (3) \\ &= \mathtt{Encode}(\mathtt{msg}) + (\mathbf{R}\mathbf{E} + \mathbf{E}'' - \mathbf{E}'\mathbf{S}) && (4) \end{aligned}$$

The recipient recovers $\mathtt{msg}$ as long as $(\mathbf{R}\mathbf{E} + \mathbf{E}'' - \mathbf{E}'\mathbf{S})$ remains small.

Exercise (solutions in the next slide)

Let us note/specify:

- a. For a matrix $\mathbf{X}$, $\|\mathbf{X}\|_{\max}$ is the max in absolute norm of its entries (or the coefficients of their entries if these are polynomials).
- b. Any unspecified matrix dimension of $\mathbf{R}, \mathbf{S}, \mathbf{E}, \mathbf{E}', \mathbf{E}''$ is n.
- c. $r = \|\mathbf{R}\|_{\max}, s = \|\mathbf{S}\|_{\max}, e = \|\mathbf{E}\|_{\max}, e' = \|\mathbf{E}'\|_{\max}, e'' = \|\mathbf{E}''\|_{\max}$

Exercise

Give a formula using $m, n, q, r, s, e, e', e''$ so that perfect correctness is guaranteed:

1. Assuming the base ring $\mathcal{R}$ is $\mathbb{Z}$, the modulus q is 2^k, $\mathtt{msg} = \{0,1\}^{n \times n}$, $\mathtt{Encode}(\mathtt{msg}) := q/2 \cdot \mathtt{msg}$ and $\mathtt{Decode}(\mathbf{M})$ rounds each coefficient of $\mathbf{M}$ to the closest value in $\{0, q/2\} \in \mathbb{Z}_q$.
2. Same conditions but the base ring $\mathcal{R}$ is $\mathbb{Z}[x]/(x^d+1)$ and $\mathtt{msg} = \{\{0,1\}^d\}^{n \times n}$.

Remember: $\mathtt{Dec}(\mathtt{msg}, \mathtt{dk}) = \mathtt{Decode}(\mathtt{Encode}(\mathtt{msg}) + (\mathbf{RE} + \mathbf{E}'' - \mathbf{E}'\mathbf{S}))$.

Solutions for the previous slide

Note that due to condition b, we have $\mathbf{S}, \mathbf{E} \in \mathcal{R}_q^{m \times n}$, $\mathbf{R}, \mathbf{E}' \in \mathcal{R}_q^{n \times m}$ and $\mathbf{E}'' \in \mathcal{R}_q^{n \times n}$.

Answer to 1

Perfect correctness is guaranteed if this value is $\leq q/4$:

$$\begin{aligned} \|\mathbf{RE} + \mathbf{E}'' - \mathbf{E}'\mathbf{S}\|_{\max} &\leq \|\mathbf{RE}\|_{\max} + \|\mathbf{E}''\|_{\max} + \|\mathbf{E}'\mathbf{S}\|_{\max} \\ &\leq mre + e'' + me's \end{aligned}$$

Answer to 2

Note that for two polynomials $f, g \in \mathbb{Z}[x]/(x^d + 1)$, we have $\|fg\|_\infty \leq d\|f\|_\infty \|g\|_\infty$. This gives the following condition on correctness:

$$d(mre + e'' + me's) \leq q/4$$

Note: these conditions are tight.

Security definitions

IND-CPA Experiment

1. $(\mathtt{ek}, \mathtt{dk}) \leftarrow \mathtt{Keygen}()$
2. $b \leftarrow \{0, 1\}$
3. $(\mathtt{msg}_0, \mathtt{msg}_1, \mathtt{st}) \leftarrow \mathcal{A}(\mathtt{ek})$
4. $\mathtt{c} \leftarrow \mathtt{Enc}(\mathtt{msg}_b, \mathtt{ek})$
5. $b' \leftarrow \mathcal{A}(\mathtt{ek}, \mathtt{c}, \mathtt{st})$
6. If $(b = b')$ return 1, else return 0.

IND-CCA Experiment

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4. $\mathtt{c} \leftarrow \mathtt{Enc}(\mathtt{msg}_b, \mathtt{ek})$
5. $b' \leftarrow \mathcal{A}^{Oracle_{\mathtt{Decaps}}(\cdot)}(\mathtt{ek}, \mathtt{c}, \mathtt{st})$
6. If $(b = b')$ return 1, else return 0.

Note: $Oracle_{\mathtt{Decaps}}(\cdot)$ is a decryption oracle for any ciphertext $\mathtt{c}' \neq \mathtt{c}$.

The advantage of an adversary $\mathcal{A}$ in either experiment is:

$$\left| \mathbb{P}[\text{The game outputs } 1] - \frac{1}{2} \right|.$$

IND-CPA security

Let us note $\texttt{Encode}(\texttt{msg}) = \mathbf{M}$. These three views are indistinguishable:

Real view: $(\mathbf{A} \text{ unif}, \mathbf{B} = \mathbf{AS} + \mathbf{E}, \mathbf{U} = \mathbf{RA} + \mathbf{E}', \mathbf{V} = \mathbf{RB} + \mathbf{E}'' + \mathbf{M})$

Hybrid 1: $(\mathbf{A} \text{ unif}, \mathbf{B} \text{ unif}, \mathbf{U} = \mathbf{RA} + \mathbf{E}', \mathbf{V} = \mathbf{RB} + \mathbf{E}'' + \mathbf{M})$
$\Leftrightarrow ([\mathbf{A}\|\mathbf{B}] \text{ unif}, [\mathbf{U}\|\mathbf{V}] = \mathbf{R}[\mathbf{A}\|\mathbf{B}] + [\mathbf{E}'\|\mathbf{E}''] + [\mathbf{0}\|\mathbf{M}])$

Hybrid 2: $([\mathbf{A}\|\mathbf{B}] \text{ unif}, [\mathbf{U}\|\mathbf{V}] \text{ unif})$

Proof outline:

- *(Real-world $\approx_c$ Hybrid 1)* under LWE
- *(Hybrid 1 $\approx_c$ Hybrid 2)* under LWE

From Blueprints to Concrete Schemes

IND-CCA security: the problem

The "Noisy El Gamal" scheme is only IND-CPA secure. Example of a CCA attack:

- Remember that $\mathbf{E}^* = (\mathbf{RE} + \mathbf{E}'' - \mathbf{E}'\mathbf{S})$ must be small in order to have $\mathtt{Decode}(\mathtt{Encode}(\mathtt{msg}) + \mathbf{E}^*) = \mathtt{msg}$.
- $\mathcal{A}$ can set $\mathbf{R} = \mathbf{0}$, $\mathbf{E}' = \mathbf{I}$,[1] and $\mathbf{E}''$ arbitrarily when computing $\mathtt{c} = \mathtt{Enc}(\mathtt{msg}, \mathtt{ek})$.
- By checking on which coefficients $\mathtt{Dec}(\mathtt{c}, \mathtt{dk})$ and $\mathtt{msg}$ differ, the attacker can learn which coefficients of $(\mathbf{E}'' - \mathbf{S})$ are "too large" and gradually recover $\mathbf{S}$.

See also *key-mismatch attacks* [DFR18, BGRR19], with an even weaker attack model.

[1] For simplicity, we assume that $\mathbf{E}'$ is square, which is e.g. the case in Ring-LWE schemes.

IND-CCA security: the solution

A generic solution: CPA-to-CCA transforms.

- *Generically* transform an IND-CPA scheme into an IND-CCA scheme.
- The resulting IND-CCA scheme is not necessarily a PKE, can also be e.g. a KEM.
- Fujisaki-Okamoto transforms [FO99a, FO99b] and their variants are the most common ones. High-level idea:
  - During encryption, generate the encryption randomness (here $\mathbf{R}, \mathbf{E}', \mathbf{E}''$) by passing $\mathtt{msg}$ (and $\mathtt{ek}$) into a PRF: $(\mathbf{R}, \mathbf{E}', \mathbf{E}'') := F(\mathtt{msg}, \mathtt{ek})$.
  - During decryption, recompute $(\mathbf{R}, \mathbf{E}', \mathbf{E}'')$ and therefore $\mathtt{c}$ from $\mathtt{msg}$ (and $\mathtt{ek}$). If they don't match, abort or output a pseudo-random shared key (in case of a KEM).

Decryption failures

Some lattice-based schemes suffer from decryption failures: a small portion of $(\mathbf{R}, \mathbf{E}', \mathbf{E}'')$ may lead to an incorrect decryption (error or different message) for dk.

- **Why do they happen?** Perfect correctness may require larger parameters.
- **Can this be an issue?** Yes:
  - Once a decryption failures is found, it is easier to find others. [DRV20]
  - Decryption failures can be exploited to mount key-recovery attacks [DGJ+19, DVV19, GJY19].
- **Note:** They can also impact code-based and rank-based schemes.
- Also work against IND-CCA: brute-force `msg` until decryption failures are found.

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- **Note:** They can also impact code-based and rank-based schemes.
- Also work against IND-CCA: brute-force $\mathtt{msg}$ until decryption failures are found.

The solution: set the probability p of decryption failures to be $\leq 2^{-\kappa}$.

- $p = 0$: NTRU [CDH+20], NTRU Prime [BBC+20]
- $p \leq 2^{-\kappa}$: Kyber [SAB+20], Saber [DKR+20], FrodoKEM [NAB+20]
- One can reduce p by embedding an error-correcting code in $\mathtt{msg}$, but it is risky:
  - Side-channel attacks [DTVV19]
  - Theoretical attacks [DVV19, GJY19]

Questions?

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