Falcon - An Update

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BROWN

What is Falcon?

- Falcon stands for:

  Fast Fourier lattice-based compact signatures over NTRU

- Falcon is a:
  - Signature scheme
  - Based on the GPV framework [GPV08]
  - Relying on NTRU lattices [HHP+03]

- The main design principle:

  Compactness: to minimize $|\mathsf{pk}| + |\mathsf{sig}|$

What's new?

What remained the same?

- Almost everything
- Specification for NIST levels I and V
- Security estimates

What changed?

- We removed the parameter set for NIST level III
  - Specification becomes much simpler
  - Algorithm count: **22** $\rightarrow$ **14**
  - Now only one modulus ($q = 12289$), one type of ring ($\mathbb{Z}[x]/(x^n + 1)$)
- New portable and constant-time implementations

Thanks to the community [OSHG19, ZSS18, KRVV19, LAZ19] for helping to improve Falcon.

Falcon in a Nutshell

We work over the cyclotomic ring $\mathcal{R} = \mathbb{Z}_q[x]/(x^n + 1)$.

→ **Keygen()**

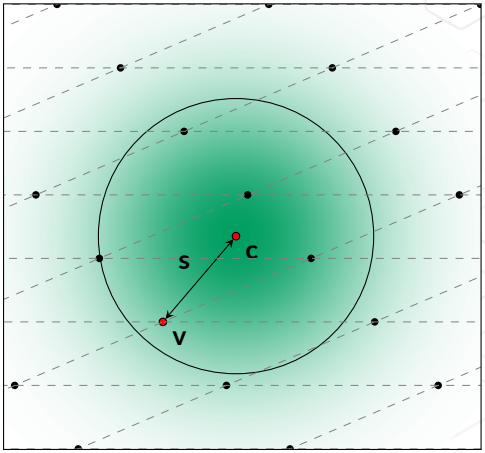
1. Gen. matrices **A**, **B** with coefficients in $\mathcal{R}$ such that:
   - **BA** $= 0$
   - **B** has small coefficients
2. pk $\leftarrow$ **A**
3. sk $\leftarrow$ **B**

→ **Sign(m, sk)**

1. Compute **c** such that **cA** $= H(\mathrm{m})$
2. **v** $\leftarrow$ "a vector in the lattice Λ(**B**), close to **c**"
3. **s** $\leftarrow$ **c** $-$ **v**

The signature sig is **s** $= (s_1, s_2)$

$\Rightarrow$



→ **Verify(m, pk, sig)**
Accept iff:

1. **s** is short
2. **sA** $= H(\mathrm{m})$

Security

On the theory side, Falcon instantiates the GPV framework:

- Tight security proof in the ROM [GPV08]
- Tight security proof in the QROM [BDF+11]

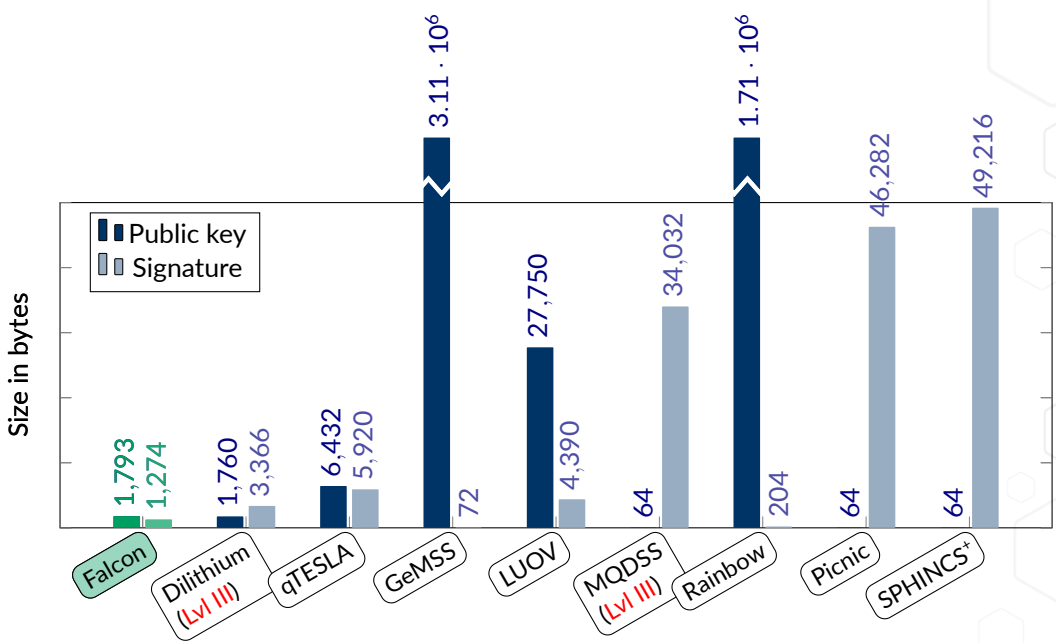
On the practical side, we consider the following lines of attack:

- **Lattice reduction** ⇒ The most effective [MW16]
- **Learning attacks** [GJSS01, GS02, NR06, DN12, YD18] ⇒ Impervious by design
- **"*Overstretched NTRU*"** [ABD16, CJL16, KF17] ⇒ Immune by parameters
- **Combinatorial** [How07, BKW00] ⇒ Immune by parameters
- **Algebraic** [CDPR16, CDW17, DPW19] ⇒ Not a threat as far as we know

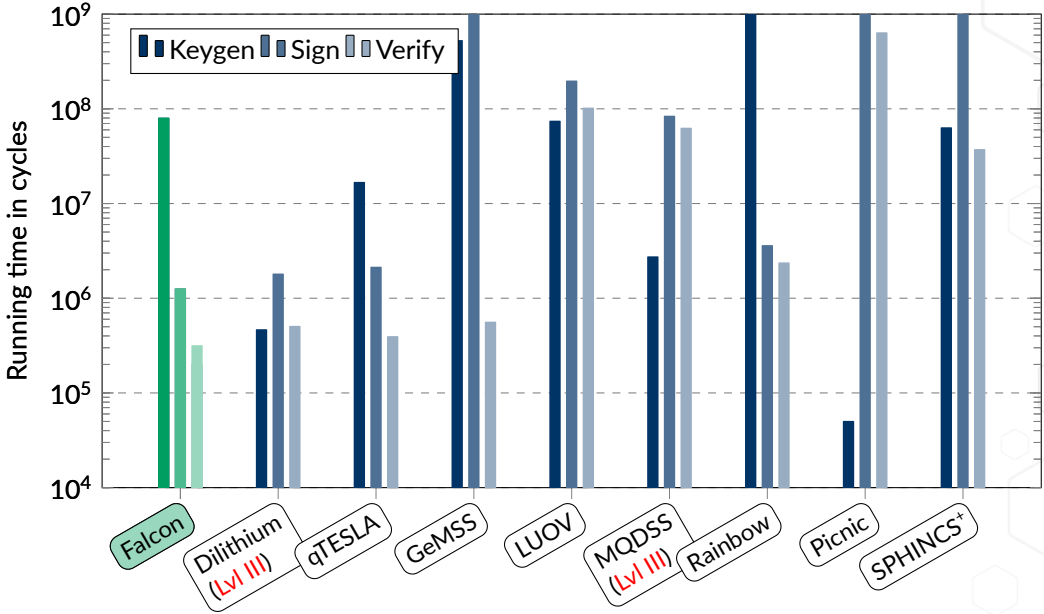
NTRU lattices:

- Extensively studied [HPS98, CS97, May99, MS01, HHPW05, GHN06, How07, Flu15]
- "Large" secrets f, g makes Falcon immune against many attacks

Communication Costs at NIST Level V (Spec.)



Computation Costs at NIST Level V (Spec.)



New Implementation(s)

- **Portable:**
  - If no FPU available, FP arithmetic is software emulated
    - Performance hit of emulation ⇒ About one order of magnitude
    - No infinites, NaNs or subnormals
  - Tested on x86/PowerPC/ARM, in 32- and 64-bit
    - Max stack < 3kB
    - Max RAM < 80 kB

- **Fully constant-time:**
  - New Gaussian sampler over the integers
    - Simple, fast, portable and constant-time
    - See Mélissa's talk this afternoon [PRR19]
  - Variable-time operations eliminated from signing procedure
  - Memory accesses only at non-secret addresses

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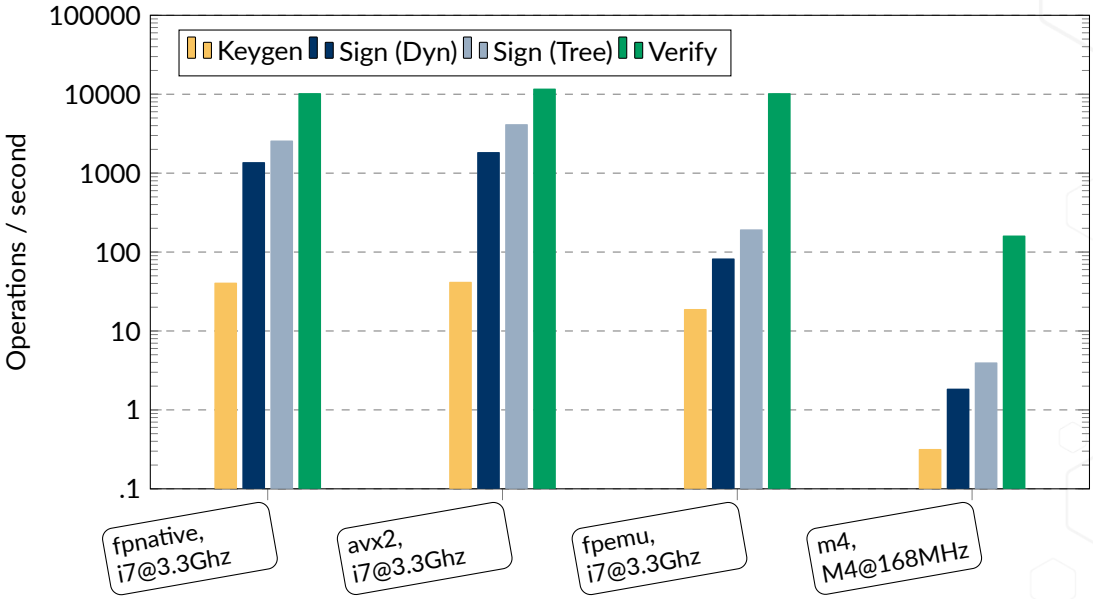
- **Integrated** to PQClean, pqm4 and SUPERCOP.
- The code and associated note are both on Falcon's website.

New Implementations - NIST Level V



Additional Features

3 modes of operation (sizes in bytes, NIST level V):

→ **Classical:**	$\|\mathsf{pk}\| = 1793$	$\|\mathsf{sig}\| = 1273$	Total = 3066
→ **Message-recovery [dLP16]:**	$\|\mathsf{pk}\| = 1793$	$\|\mathsf{sig}\| = 768^*$	Total = 2561
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Falcon can be turned into an IBE (identity-based encryption) scheme:

Falcon + New Hope = IBE

- → See [GPV08, DLP14, MSO17] for details
- → Orders of magnitude faster than pairing-based IBEs

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Falcon can also be turned into a ring signature scheme (variation of [RST01], [LAZ19]).

Conclusion

Falcon is still:

- Secure
- Compact
- Fast
- Modular (3 modes, IBE, etc.)

Falcon is now:

- Simpler
- Portable
- Constant-time

Use cases:

- Certificate authorities
- Blockchain
- Firmware update
- IBE
- Ring signatures

The future:

- New, unique functionalities
- Sanity check: statistical test suite

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Thank you!

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