Lattice-based NIST Candidates

Abstractions and Ninja Tricks

Thomas Prest

PQShield

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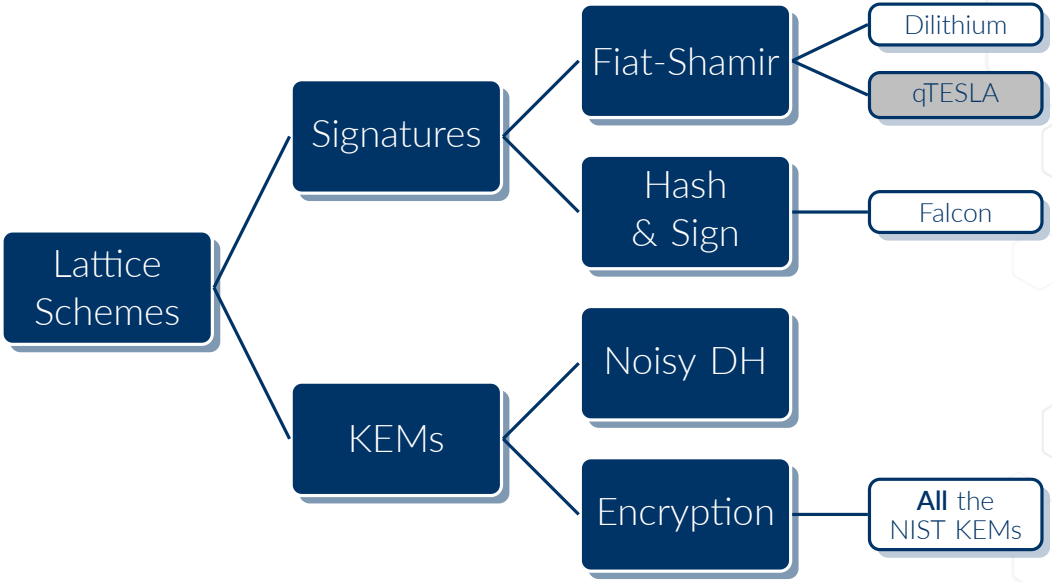
Introduction


Lattice Schemes
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KEMs
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Noisy DH
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Dilithium
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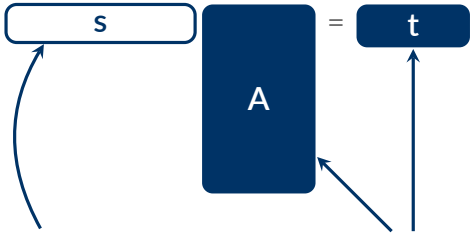
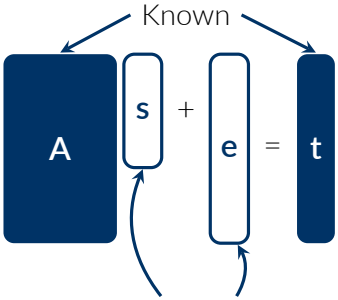
All the NIST KEMs

Module-SIS



Module-LWE



NTRU

Find small f, g such that $g \cdot f^{-1} = h$ in $\mathcal{R}_q = \mathbb{Z}_q[x]/(\varphi)$

Key Encapsulation Mechanisms

Keygen$(\mathbf{A} \in \mathcal{R}_q^{m \times m})$

1. $\mathbf{S}, \mathbf{E} \leftarrow \chi_1 \times \chi_2$
2. $\mathbf{B} \leftarrow \mathbf{AS} + \mathbf{E}$
3. $\mathtt{sk} := (\mathbf{S}, \mathbf{E}), \mathtt{pk} := \mathbf{B}$

Enc$(\mathtt{M}, \mathbf{pk})$

1. $\mathbf{R}, \mathbf{E}', \mathbf{E}'' \leftarrow \chi_3 \times \chi_4 \times \chi_5$
2. $\mathbf{U} \leftarrow \mathbf{RA} + \mathbf{E}'$
3. $\mathbf{V} \leftarrow \mathbf{RB} + \mathbf{E}'' + \mathrm{Encode}(\mathtt{M})$
4. $\mathtt{ct} := (\mathbf{U}, \mathbf{V})$

Dec$(\mathbf{ct}, \mathbf{sk})$

1. $\mathtt{M} \leftarrow \mathrm{Decode}(\mathbf{V} - \mathbf{US})$

Think of El Gamal, but with LWE/LWR.

Damien's excellent talk covers NTRU: https://youtu.be/yZhmKwmx48o

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Underlying problem/lattice

- → LWE
- → Module-LWE
- → Ring-LWE
- → (Module-)LWR
- → (Module-)Integer-LWE
- → and also NTRU (not here)

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Dec$(\mathtt{ct}, \mathbf{sk})$

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Underlying ring $\mathcal{R}_q$

- $\mathbb{Z}_q$, q small
- $\mathbb{Z}_q[x]/(x^n + 1)$, q prime
- $\mathbb{Z}_q[x]/(x^n + 1)$, q power-of-2
- $\mathbb{Z}_q[x]/(P)$, $P = x^p - x - 1$ irreducible mod q, and q prime
- $\mathbb{Z}_q$, q huge

Keygen$(\mathbf{A} \in \mathcal{R}_q^{m \times m})$

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Distributions χ_i

- Binomial
- Pseudo-Gaussian
- Small uniform
- Ternary (NTRU Prime, NTRU)
- No error distribution (LWR)

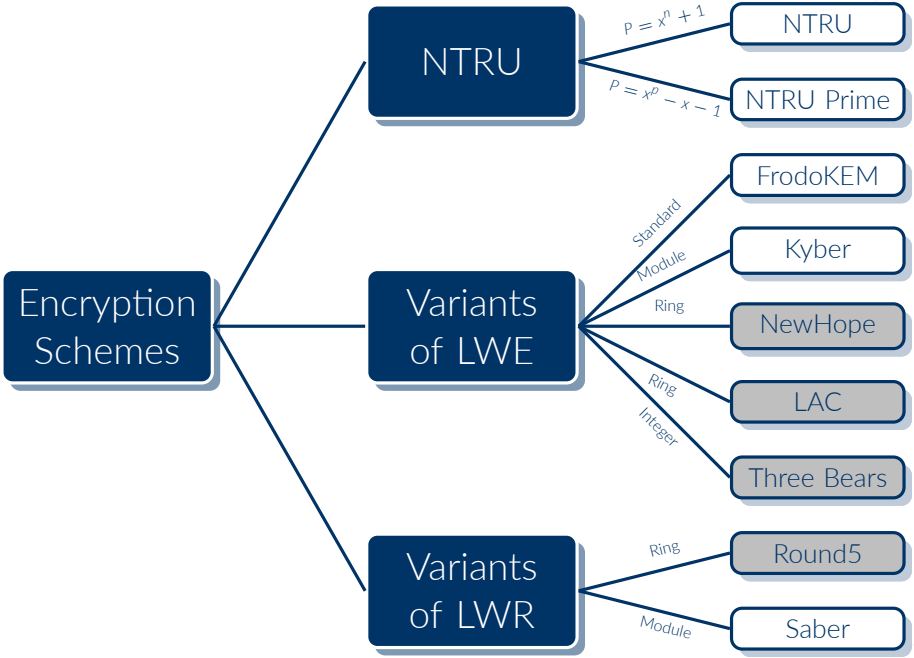
Obtaining security against active attackers

Most schemes rely on transforms in [HHK17]:

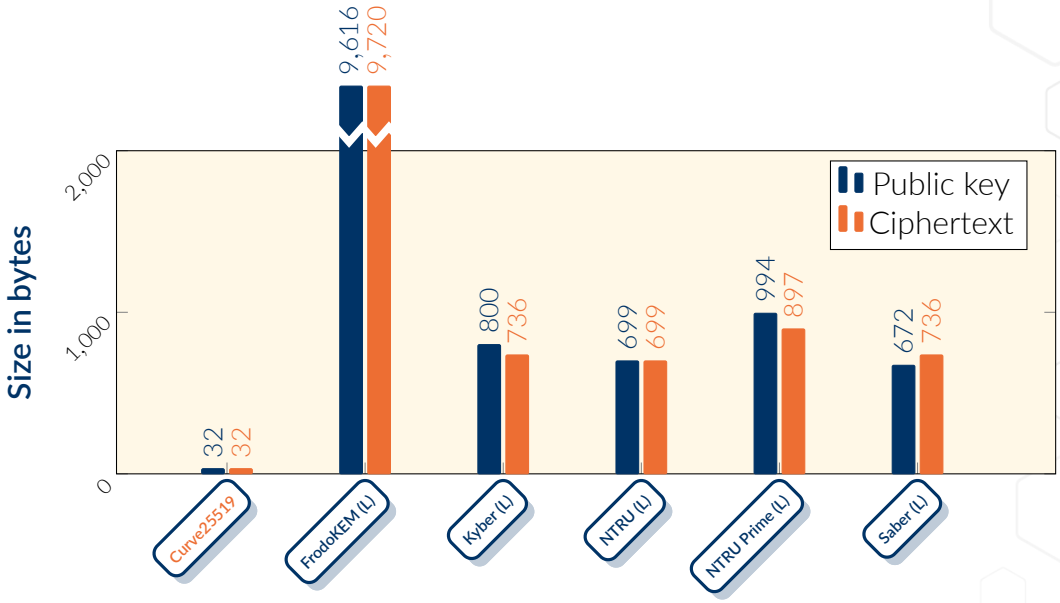
- Variants of Fujisaki-Okamoto, handle decryption failures [DRV20]
- Tight proofs in the ROM but not the QROM

NTRU and NTRU Prime use [BP18] and [Den03] instead:

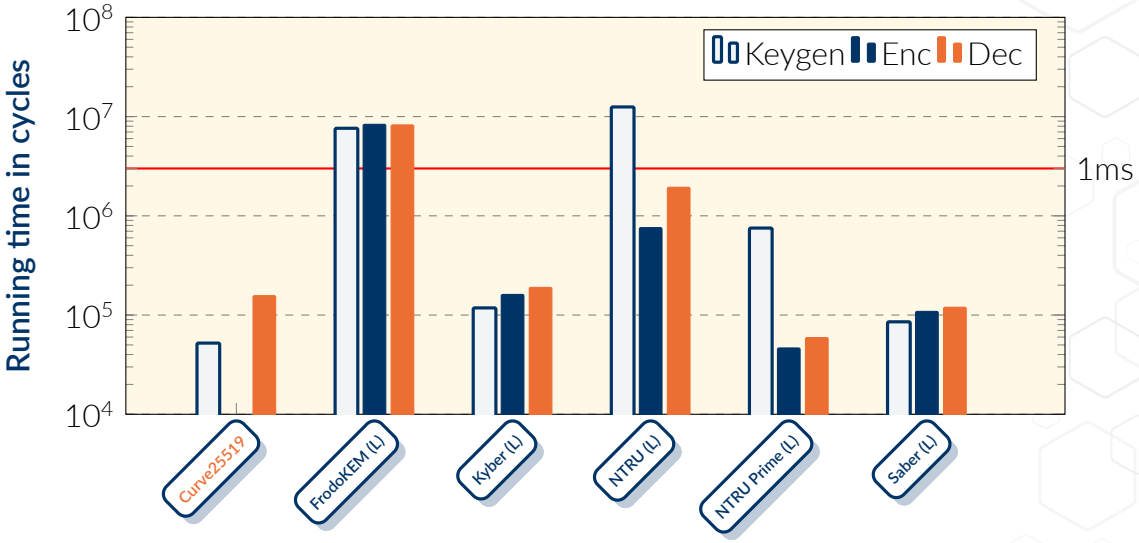
- Do not require re-encryption
- Tight proofs in the QROM (under non-standard assumptions)


Encryption Schemes
NTRU
$p = x^n + 1$
NTRU
$P = x^p - x - 1$
NTRU Prime
Variants of LWE
Standard
FrodoKEM
Module
Kyber
Ring
NewHope
Ring
LAC
Integer
Three Bears
Variants of LWR
Ring
Round5
Module
Saber

Bandwidth cost of Level 1 KEMs

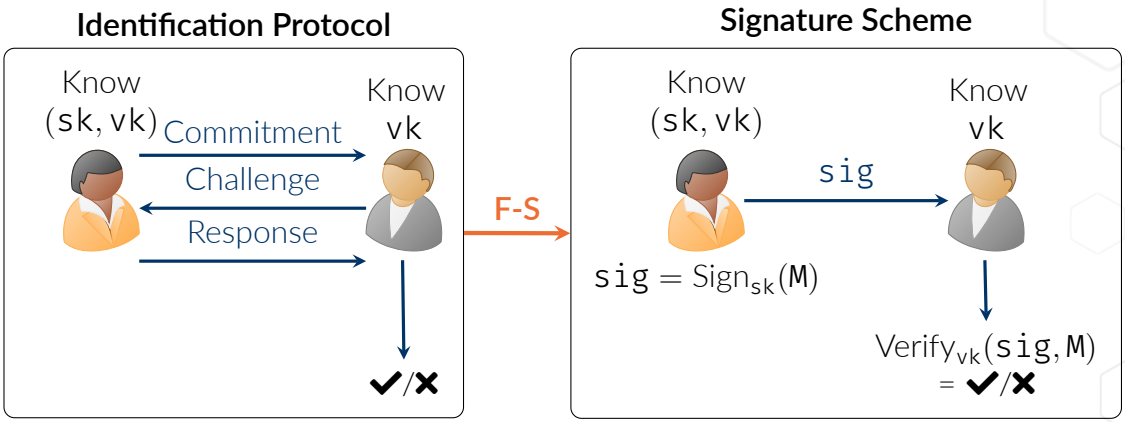


Computation Cost of Level 1 KEMs



Signatures

Fiat-Shamir Signatures



F-S refers to the Fiat-Shamir transform:

- The challenge is now defined as $H(\text{Commitment}\|\mathtt{M})$.
- The signature is (Commitment,Response).

Fiat-Shamir w/ discrete log

Keygen$(g \in G)$

1. $x \leftarrow \mathbb{Z}_q^\times$
2. $h \leftarrow g^x$
3. $\mathtt{sk} := x, \mathtt{pk} := h$

Sign$(\mathtt{M}, \mathbf{sk})$

1. $r \leftarrow \mathbb{Z}_q^\times$
2. $u \leftarrow g^r$
3. $c \leftarrow H(u \| \mathtt{M})$
4. $z \leftarrow r - cx$
5. $\mathtt{sig} := (c, z)$

Verify$(\mathtt{M}, \mathbf{pk})$

1. Accept if and only if $H(g^z \cdot h^c \| \mathtt{M}) = c$

3 crucial properties of ID protocol:

1. **Correctness:**
   An honest prover can convince a verifier he knows sk
2. **Soundness:**
   A dishonest prover cannot convince a verifier he knows sk
3. **(Honest Verifier) Zero-Knowledge:**
   No information about sk *is leaked*

Virtually all lattice-based Fiat-Shamir schemes transpose this blueprint to lattices, with 3 tricks:

- Rejection sampling (a.k.a. *Fiat-Shamir with aborts*)
- The Bai-Galbraith trick [BG14]
- The Dilithium trick [LDK+17]

Keygen$(\mathbf{A} \in \mathcal{R}_q^{k \times \ell})$

1. $\mathbf{s}_1, \mathbf{s}_2 \leftarrow \chi_1 \times \chi_2$ (short)
2. $\mathbf{t} \leftarrow \mathbf{A}\mathbf{s}_1 + \mathbf{s}_2$
3. $\mathtt{sk} := (\mathbf{s}_1, \mathbf{s}_2), \mathtt{pk} := \mathbf{t}$

Sign$(\mathtt{M}, \mathbf{sk})$

1. $\mathbf{r}_1, \mathbf{r}_2 \leftarrow \chi_3 \times \chi_4$ (short)
2. $\mathbf{u} \leftarrow \mathbf{A}\mathbf{r}_1 + \mathbf{r}_2$
3. $\mathbf{c} \leftarrow H(\mathbf{u} \| \mathtt{M})$ (short)
4. $\mathbf{z}_1 \leftarrow \mathbf{r}_1 - \mathbf{c}\mathbf{s}_1$
5. $\mathbf{z}_2 \leftarrow \mathbf{r}_2 - \mathbf{c}\mathbf{s}_2$
6. Rej. sampling (for HVZK)
7. $\mathtt{sig} := (\mathbf{c}, \mathbf{z}_1, \mathbf{z}_2)$

Verify$(\mathtt{M}, \mathbf{pk})$

1. Accept iff $(\mathbf{z}_1, \mathbf{z}_2)$ is short and $H(\mathbf{A}\mathbf{z}_1 + \mathbf{z}_2 - \mathbf{t}\mathbf{c} \| \mathtt{M}) = \mathbf{c}$

Keygen($\mathbf{A} \in \mathcal{R}_q^{k \times \ell}$)

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Bai-Galbraith and Dilithium tricks:
Only care about most significant bits (MSB).

Bai-Galbraith trick: Discard commitment's LSBs.
$\Rightarrow$ Shorter signatures.

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5. $\mathbf{h} \leftarrow \mathrm{MSB}(\mathbf{A}\mathbf{z} - \mathbf{t}\mathbf{c}) \oplus \mathrm{MSB}(\mathbf{u})$
6. Rej. sampling + check $\mathbf{h}$ short
7. $\mathtt{sig} := (\mathbf{c}, \mathbf{z}, \mathbf{h})$

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$\Rightarrow$ Shorter public key, slightly larger signatures.

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7. $\mathtt{sig} := (\mathbf{c}, \mathbf{z}, \mathbf{h})$

Verify($\mathtt{M}$, $\mathbf{pk}$)

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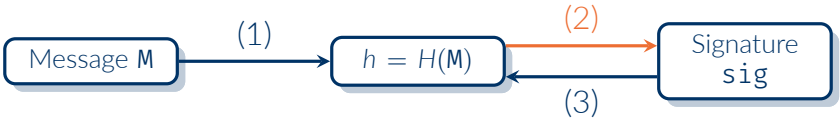
In the Dilithium trick, it is **vital** to control the norm and Hamming weight of $\mathbf{h}$. Otherwise forgery is simple:

1. Sample random $\mathbf{z}$ and $\mathbf{u}$
2. $\mathbf{c} \leftarrow H(\mathrm{MSB}(\mathbf{u}) \| \mathtt{M})$
3. $\mathbf{h} \leftarrow \mathrm{MSB}(\mathbf{A}\mathbf{z} - \mathbf{t}\mathbf{c}) \oplus \mathrm{MSB}(\mathbf{u})$

See qTESLA-*-s [BAA+19].

Signatures based on *Hash-then-Sign*



- **The signer** computes (1), then (2) using the signing key sk.
- **The verifier** computes (1), then (3) using the verification key pk, and checks that the results match.

In RSA signatures, (2) + (3) define a trapdoor permutation, but lattices rely on weaker notions: TPSF and average TPSF.

Trapdoor permutation ⇒ TPSF ⇒ Average TPSF

GPV signatures (2008)

Falcon instantiates this blueprint.

Keygen(1^λ)

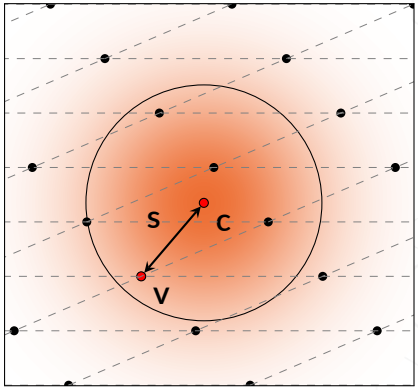
1. Gen. matrices $\mathbf{A}, \mathbf{B}$ s.t.:
   - $\mathbf{B} \cdot \mathbf{A} = 0$
   - $\mathbf{B}$ has small coefficients
2. $\mathtt{pk} := \mathbf{A}$, $\mathtt{sk} := \mathbf{B}$

Sign$(\mathtt{M}, \mathbf{sk} = \mathbf{B})$

1. Compute $\mathbf{c}$ such that $\mathbf{c} \cdot \mathbf{A} = H(\mathtt{M})$
2. $\mathbf{v} \leftarrow$ vector in $\mathcal{L}(\mathbf{B})$, close to $\mathbf{c}$
3. $\mathtt{sig} := \mathbf{s} = (\mathbf{c} - \mathbf{v})$

Verify$(\mathtt{M}, \mathbf{pk} = \mathbf{A}, \mathtt{sig} = \mathbf{s})$

Check ($\mathbf{s}$ short) & ($\mathbf{s} \cdot \mathbf{A} = H(\mathtt{M})$)



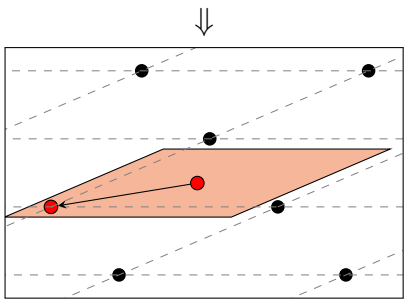
How to compute efficiently a close vector (the second algorithm assumes we precomputed the Gram-Schmidt orthogonalization $\mathbf{B} = \mathbf{L} \cdot \tilde{\mathbf{B}}$).

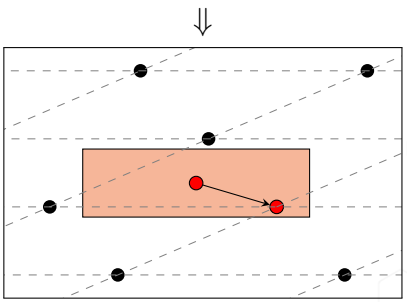
RoundOff$(\mathbf{B}, \mathbf{c})$

1. $\mathbf{t} \leftarrow \mathbf{c} \cdot \mathbf{B}^{-1}$
2. For $j \in \{n, \ldots, 1\}$:
   1. $z_j \leftarrow \lceil t_j \rfloor$
3. Return $\mathbf{v} := \mathbf{z} \cdot \mathbf{B}$

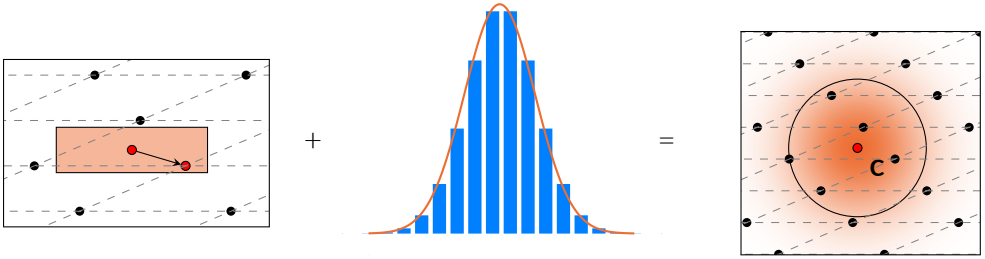
NearestPlane$(\mathbf{B}, \mathbf{L}, \mathbf{c})$

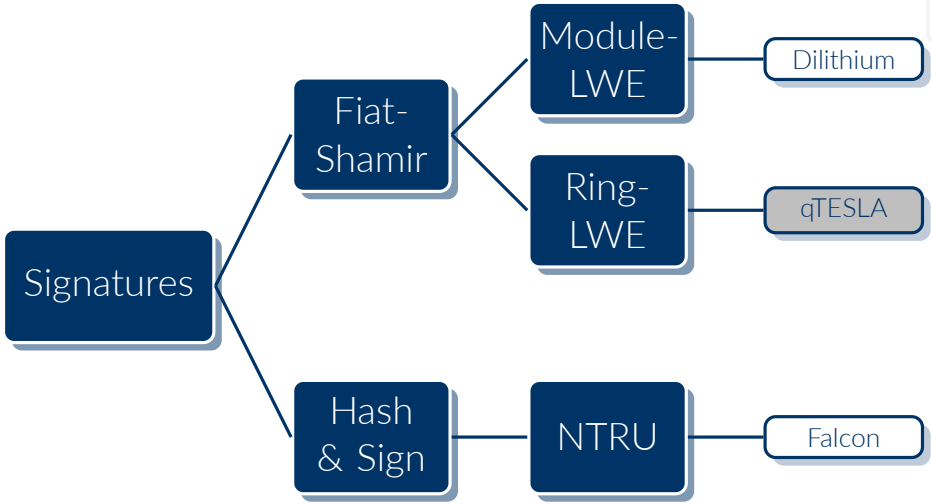
1. $\mathbf{t} \leftarrow \mathbf{c} \cdot \mathbf{B}^{-1}$
2. For $j \in \{n, \ldots, 1\}$:
   1. $z_j \leftarrow \lceil t_j + \sum_{i>j}(t_1 - z_i)L_{i,j} \rfloor$
3. Return $\mathbf{v} := \mathbf{z} \cdot \mathbf{B}$





- **Problem:** When used for signing, the algorithms RoundOff and NearestPlane leak the shape of the private key **B**, leading to attacks.
- **Solution:** Replace rounding with (Gaussian) randomized rounding.

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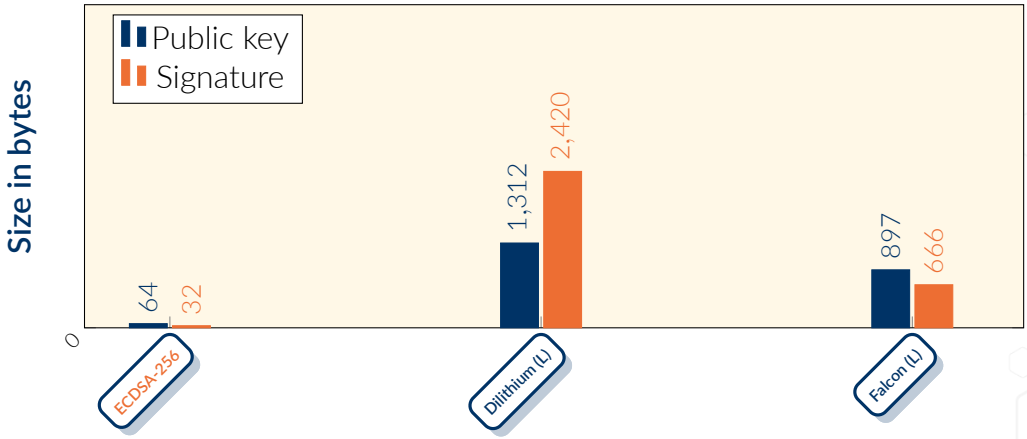
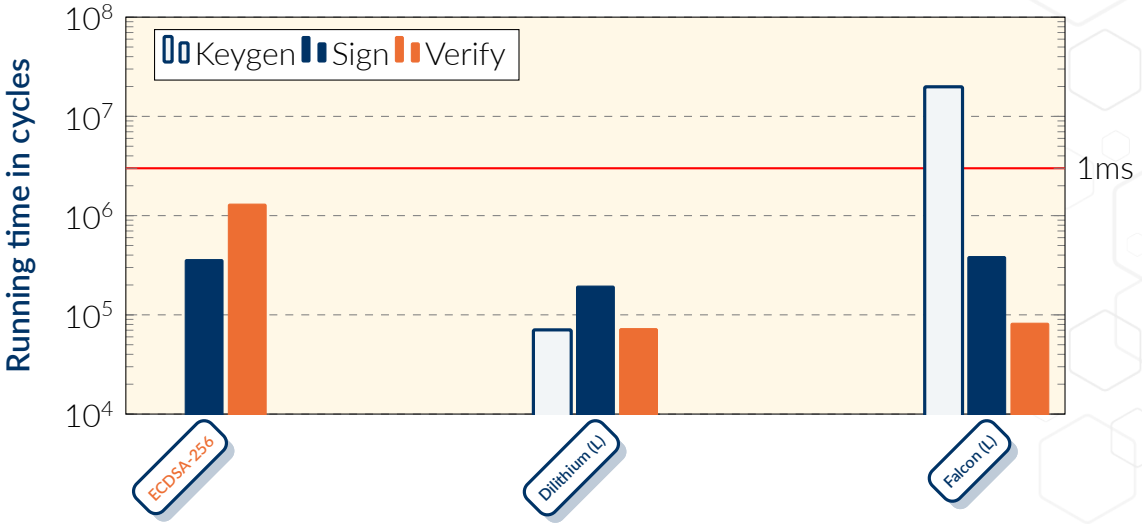
Bandwidth cost of Level 1 Signatures



Computation Cost of Level 1 Signatures



Ninja Tricks

Figure 1: Broadcast

Figure 2: Group Messaging

In LWE/LWR proposals, $\mathbf{U}$ does almost not depend on the public key.

- → Use the same $\mathbf{A}$ for all public keys.
- → Use the same $\mathbf{U}$ when encrypting **the same $\mathtt{M}$** to several recipients.

$\mathsf{Enc}(\mathtt{M}, \mathbf{pk} = (\mathbf{A}, \mathbf{B}))$

1. $\mathbf{R}, \mathbf{E}', \mathbf{E}'' \leftarrow \chi_3 \times \chi_4 \times \chi_5$
2. $\mathbf{U} \leftarrow \mathbf{RA} + \mathbf{E}'$
3. $\mathbf{V} \leftarrow \mathbf{RB} + \mathbf{E}'' + \mathsf{Encode}(\mathtt{M})$
4. $\mathtt{ct} := (\mathbf{U}, \mathbf{V})$

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4. $\mathtt{ct} := (\mathbf{U}, \mathbf{V})$

$\Longrightarrow$

$\mathsf{MultiEnc}(\mathtt{M}, \mathbf{pk}_1, \ldots, \mathbf{pk}_k)$

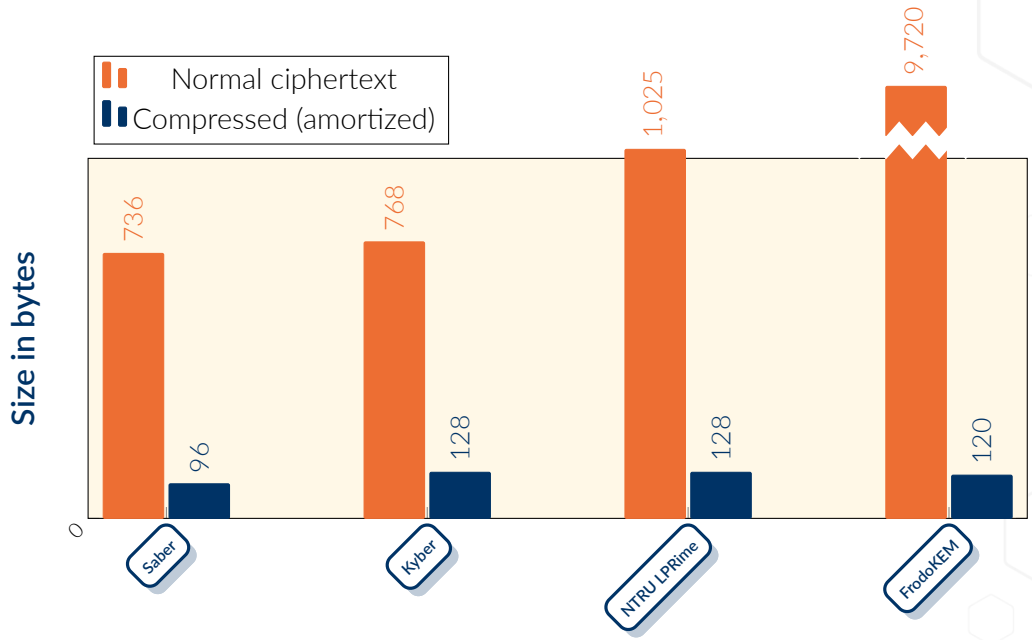
1. $\mathbf{R}, \mathbf{E}' \leftarrow \chi_3 \times \chi_4$
2. $\mathbf{U} \leftarrow \mathbf{RA} + \mathbf{E}'$
3. For $i = 1, \ldots, k$:
   1. $\mathbf{E}_i'' \leftarrow \chi_5$
   2. $\mathbf{V}_i \leftarrow \mathbf{RB}_i + \mathbf{E}_i'' + \mathsf{Encode}(\mathtt{M})$
4. $\mathtt{ct} := (\mathbf{U}, \mathbf{V}_1, \ldots, \mathbf{V}_k)$

This improves amortized costs by **factors up to 169**.

- → Faster encryption
- → Smaller ciphertexts

See [KKPP20].

Impact on Potential NIST Standards (Level I)

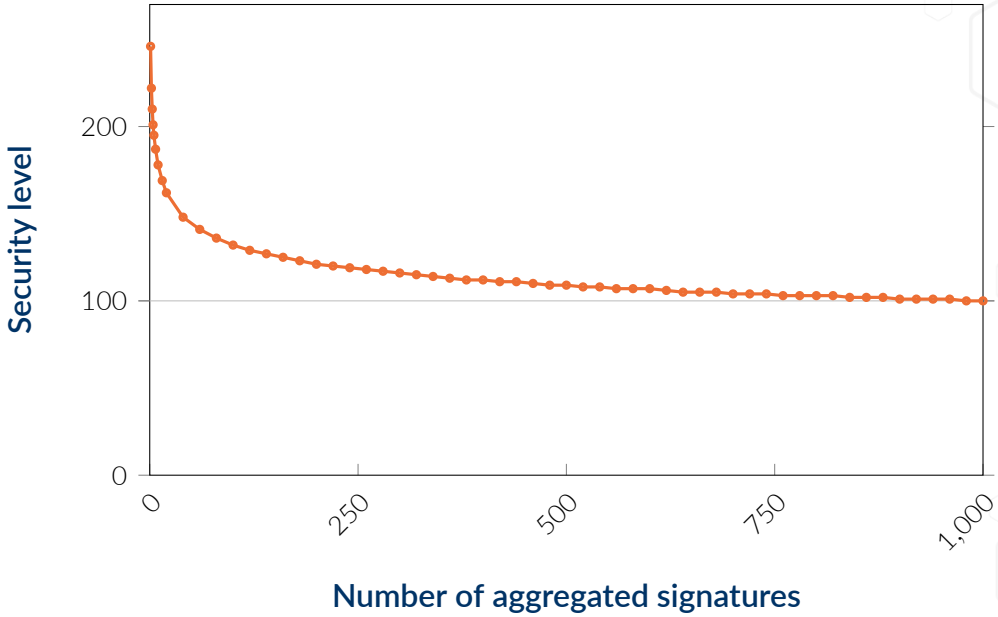


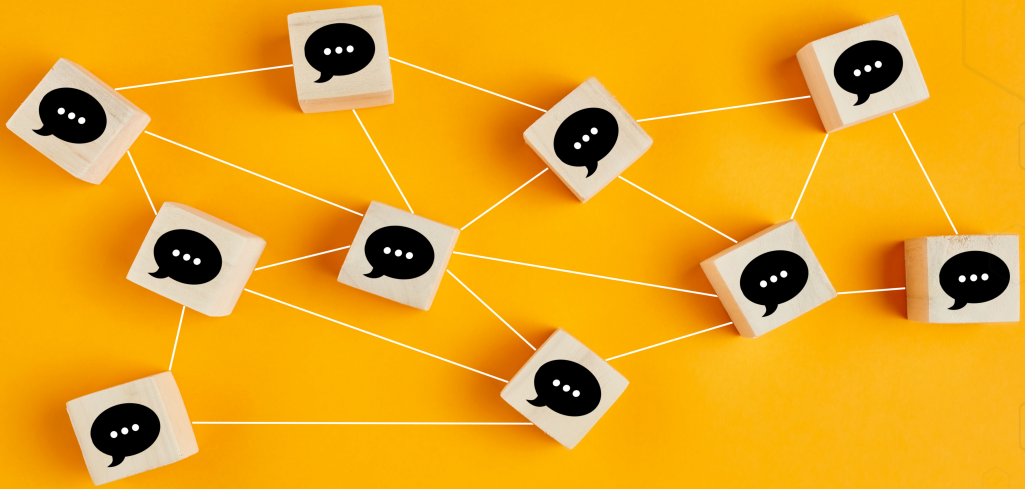
A cute property of GPV signatures

Like all GPV signatures, Falcon supports *single-signer* signature aggregation.

$$\left.\begin{aligned} \mathbf{s}_1 \cdot \mathbf{A} &= H(\mathtt{M}_1) \\ &\vdots \\ \mathbf{s}_k \cdot \mathbf{A} &= H(\mathtt{M}_k) \end{aligned}\right\} \Longrightarrow \left(\sum_i \mathbf{s}_i\right) \cdot \mathbf{A} = \sum_i H(\mathtt{M}_i)$$

For up to **1000 signatures**, aggregate signature size **< 3kB**.


Security level
200
100
0
0
250
500
750
1,000
Number of aggregated signatures



Questions?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